Abrupt switching from hadron gas to quark-gluon plasma at smooth chiral crossover

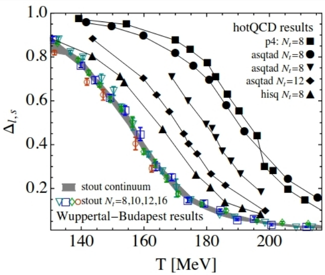
Oleksii Ivanytskyi

D. Blaschke, M. Cierniak, OI, G. Röpke, EPJ A (2024)
OI, D. Blaschke, G. Röpke, J.Subatom.Part.Cosm. (2025)
D. Blaschke, OI, G. Röpke, PoS QCHSC24 247 (2025)

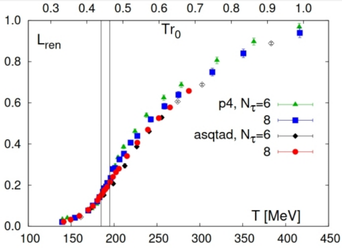
XVII Polish Workshop on Relativistic Heavy Ion Collisions

Kraków, 13-14 December 2025

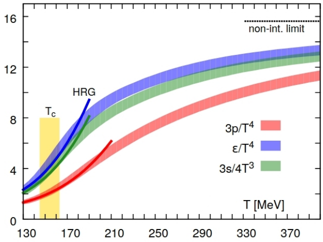
Smooth order parameters and EoS



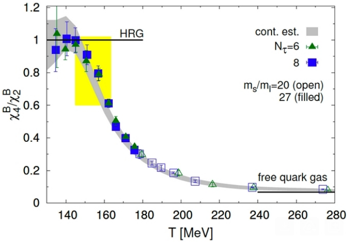
Bazavov et al., PRD (2014)



Bazavov et al., PRD (2017)



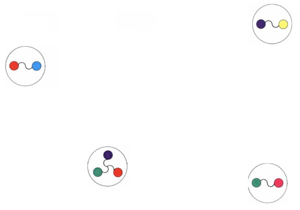
Bazavov et al., JHEP (2010)



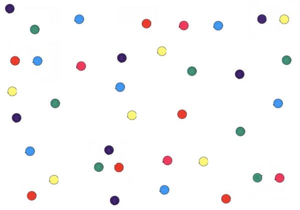
Bazavov et al., PRD (2009)

(De)confinement phase transition

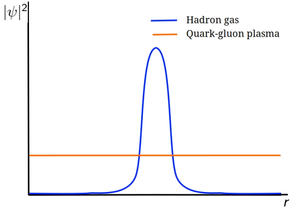
Hadronic gas



Quark-gluon plasma

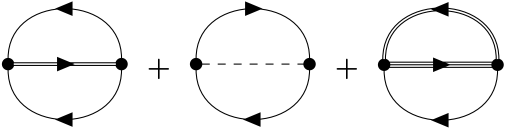


Can a catastrophic rearrangement of ψ lead to a smooth thermodynamics?



A unified quark-hadron approach is needed

Cluster decomposition in two-loop approximation

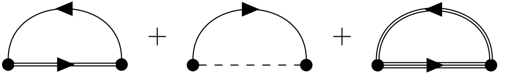
$\varphi =$  $+ \ldots$

diquarks **mesons** **baryons**

G. Baym, L.P. Kadanoff, Phys. Rev. 124, 287 (1961); G. Baym, Phys. Rev. 127, 1391 (1962)

- **Two-loop self-energies & Dyson-Schwinger propagators**

$$\Pi_n = \frac{\delta\varphi}{\delta S_n}, \quad (S_n)^{-1} = (S_n^{free})^{-1} - \Pi_n, \quad n = 1, 2, 3, \ldots$$

quarks: $\Pi_1 =$  $+ \ldots$

Dyson-Schwinger problem requires solving all S_n simultaneously

- **Mean-field approximation for quark propagators**

The Dyson-Schwinger problem reduces to a subsequent solving S_n using $S_{m<n}$

QCD thermodynamic potential

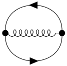
$$\Omega_{\rm QCD} = \underbrace{\Omega_{quarks} + \overbrace{\mathcal{U}_{\chi} - \langle \overline{q}\hat{\Sigma} q\rangle}^{\chi-\textbf{condensates}}}_{\chi-\textbf{symmetric density functional}} + \mathcal{U}_{\rm gluons} + \underbrace{\Omega_{\rm hadrons} + \Omega_{\rm colored\ clusters}}_{\substack{\textbf{multiquark clusters within}\\ \textbf{Beth}-\textbf{Uhlenbeck approach}}}$$

- **Quarks**
  - Non-perturbative states at low momenta $k < \Lambda$, S_{quarks} - mean-field propagator

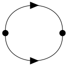
$$\Omega^{k<\Lambda}_{quarks} = -\frac{1}{\beta V} Tr \ln(\beta S^{-1}_{quarks})$$

  - Perturbative states at high momenta $k > \Lambda$

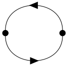
$$\Omega^{k>\Lambda}_{quarks} = \frac{1}{2\beta V}$$



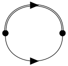
- **Confinement mechanism** – chirally symmetric density functional
- **Gluons** – Polyakov-loop potential
- **Hadrons** - 62 mesons, 60+60 (anti)baryons states with $M < 2.6$ GeV
- **Colored multiquark states** - diquarks, tetraquarks, pentaquarks coupled to Φ



diquark



meson



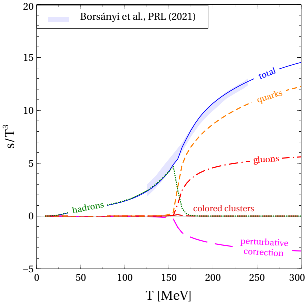
baryon

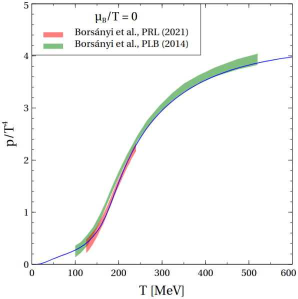
EoS

$$s = -\frac{\partial \Omega}{\partial T}$$

$$p = -\Omega + \Omega_{vacuum} = \int dTs$$

- **Low T**
  1. hadron dominance
- **High T**
  1. quark-gluon dominance
  2. negative perturbative contribution
- **Colored multiquark states**
  1. suppressed by the Polyakov loop at high T
  2. suppressed by high mass at high T





Composition

$$s_i = -\frac{\partial \Omega_i}{\partial T}$$

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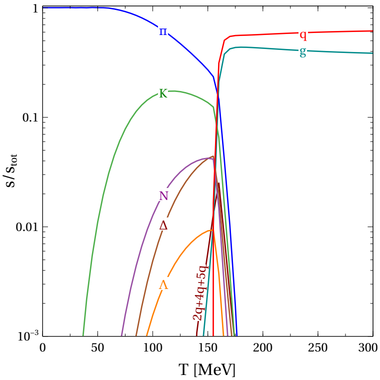

Sharp switching between partonic & hadronic degrees of freedom

Speed of sound

$$c_S^2 = \frac{\partial p}{\partial \varepsilon}, \quad \varepsilon = Ts - p, \quad c_{Si}^2 = \frac{\partial p_i}{\partial \varepsilon_i}$$

- **Low T**
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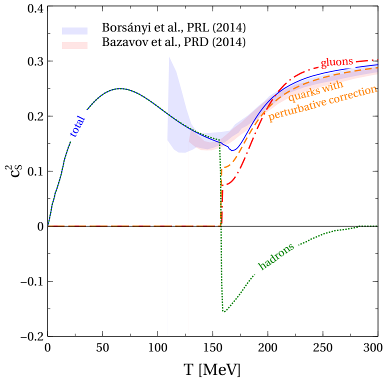


Mechanical instability of hadron component above T_c requires quarks & gluons

Expansion & reaction rates

- "Hubble" expansion of the adiabatic fireball

$$sV = const \quad \Rightarrow \quad H_{exp} = \frac{1}{\tau_{exp}} = \frac{s^{1/3}}{a}$$

- Reaction rate (pion-kaon-hadron system)

$$\tau_i^{-1} = \sum_j \sigma_{ij} n_j$$

  - Hadron-hadron scatterings

$$\sigma_{ij} = \lambda \langle r_i^2 \rangle \langle r_j^2 \rangle, \quad \lambda = 0.197 \mathrm{GeV}^2$$

B. Povh and J. Hüfner, Phys. Lett. B 245 (1990); J. Hüfner and B. Povh, Phys. Rev. D 46 (1992)

  - Quark-hadron scatterings

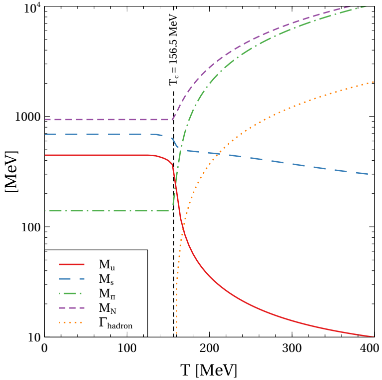
$$\sigma_{iq} = \pi \langle r_i^2 \rangle$$

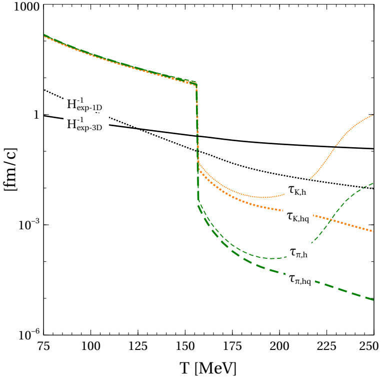
  - Mean squared radii of hadrons

$$\langle r_i^2 \rangle = \frac{3}{4\pi^2 f_i^2}, \quad f_i^2 = \frac{1}{2M_i^2} \left(N_i^l \langle \bar{l} l \rangle + N_i^s \langle \bar{s} s \rangle \right) \left(N_i^l m_l + N_i^s m_s \right), \quad \langle \bar{f} f \rangle \propto m_f^*$$

H.J. Hippe and S.P. Klevansky, Phys. Rev. C 52 (1995)

Universal chemical freeze-out





- **Low T:** χ-broken $\Rightarrow$ heavy quark $\Rightarrow$ "small" hadrons $\Rightarrow$ small reaction rate
- **High T:** χ-restored $\Rightarrow$ light quark $\Rightarrow$ "large" hadrons $\Rightarrow$ large reaction rate

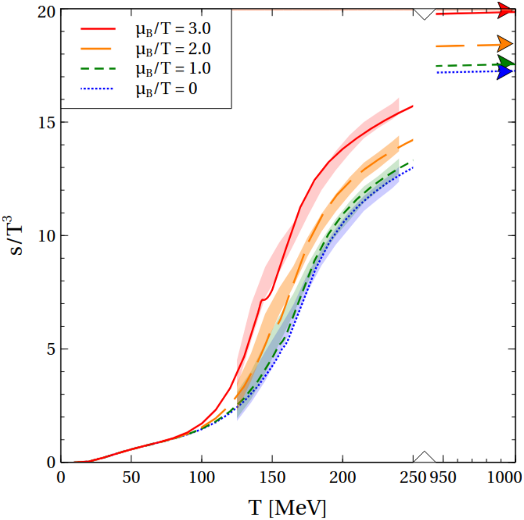
Abrupt Mott dissociation of hadrons causes universal free-out

Conclusions

- A phenomenological "confinement" mechanism (details are skipped)
- A unified EoS of QCD matter based on a cluster decomposition approach
- Agreement with the lattice QCD data and

  the astrophysical data on neutron stars (not shown)
- Sudden switching between partonic and hadronic degrees of freedom
- Universal chemical freeze-out due to abrupt Mott dissociation

Generalization to finite baryon asymmetry ($\mu_B \neq 0$)



Chiral condensate

$$\langle \bar{f} f \rangle = -\frac{\partial \Omega}{\partial m_f} = \underbrace{2N_c \int \frac{d\mathbf{k}}{(2\pi)^3} \frac{M_f}{E_f} (f_f^+ + f_f^- - 1)}_{\text{quarks}}$$

$$+ \underbrace{\sum_{n>1} \frac{d_n \sigma_n^f}{m_f} \int \frac{d\mathbf{k}}{(2\pi)^3} \int \frac{d\omega}{\pi} \frac{M_n}{\omega} (f_n^+ + f_n^-) \sin^2 \delta_n \frac{\partial \delta_n}{\partial \omega}}_{\text{multiquark clusters}}$$

- **σ-factor**
  1. σ_π^f, σ_K^f – defined from the GMOR relations
  2. other multiquark clusters

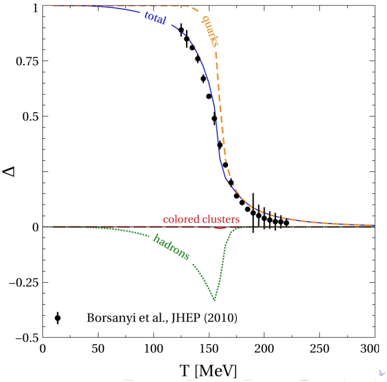
$$\sigma_n^f = m_f \frac{\partial M_n}{\partial m_f} = m_f N_n^f$$

J. Jankowski, D. Blaschke, M. Spalinski, PRD 87, 10 (2013)

- **Scaled chiral condensate**

$$\Delta = \frac{m_s \langle \bar{l} l \rangle - m_l \langle \bar{s} s \rangle}{m_s \langle \bar{l} l \rangle_0 - m_l \langle \bar{s} s \rangle_0}$$

Almost constant quark term below T_c
Hadrons are necessary to reproduce the lQCD data



Cumulants and composition

$$\frac{p}{T^4} = \sum_n \frac{1}{n!}\left(\frac{\mu_B}{T}\right)^n \chi_n^B, \quad \chi_n^B = T^{n-4}\frac{\partial^4 p}{\partial \mu_B^4}$$

- **Boltzmann hadron gas**

$$\frac{\chi_4^B}{\chi_2^B} = 1$$

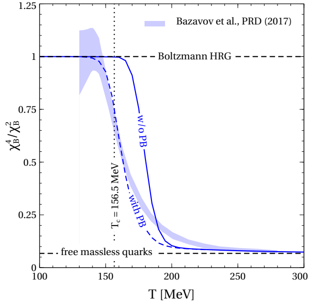
- **Free massless quarks**

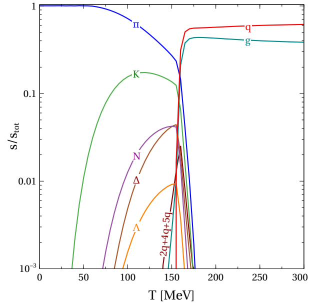
$$\frac{\chi_4^B}{\chi_2^B} = \frac{1}{N_c^2} \cdot \frac{6}{\pi^2}$$

- χ_4^B / χ_2^B **probes the quark-to-baryon ratio?**

Only asymptotically

Evidences a repulsive interaction among baryons





Confining density functional @ $N_f = 2$

$$\mathcal{L} = \overline{q}(i\partial\!\!\!/ - m)q - \mathcal{U}, \quad \mathcal{U} = D_0\left[(1+\alpha)\langle\overline{q}q\rangle_0^2 - (\overline{q}q)^2 - (\overline{q}i\vec{\tau}\gamma_5 q)^2\right]^{\varkappa}$$

O.Ivanytskyi, D. Blaschke, PRD 105, 114042 (2022)

D_0 - coupling, controls interaction strength

α - dimensionless constant, controls vacuum quark mass

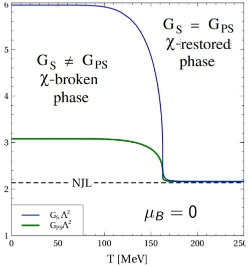
$\langle\overline{q}q\rangle_0$ - χ-condensate in vacuum (introduced for the sake of convenience)

- **Comparison to the NJL model**
  - $\varkappa = 1$: NJL model
  - $\varkappa = 1/3$: String Flip model

  mean-filed self-energy $\Sigma = \frac{\partial\mathcal{U}}{\partial\langle q^+q\rangle} \propto \overbrace{\langle q^+q\rangle}^{\text{separation}}{}^{-1/3}$

  C.J.Horowitz, E.J. Moniz, J. W. Negele, PRD 31, 1689 (1985)

  G. Röpke, D. Blaschke, H. Schultz, PRD 34, 3499 (1986)

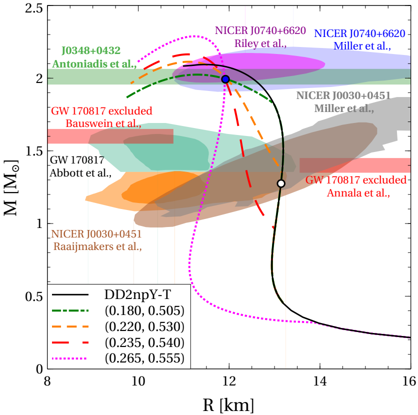


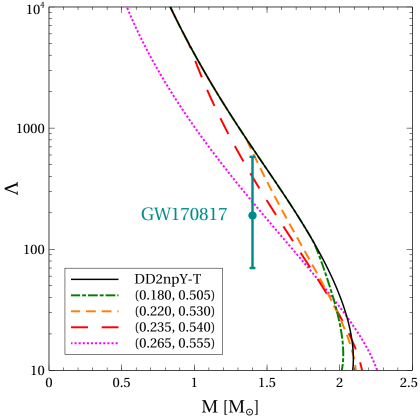
- **Dimensionality**

$$[\mathcal{U}] = energy^4 \qquad [\overline{q}q] = energy^3 \quad \Rightarrow \quad [D_0]_{\varkappa=1/3} = energy^2 = \left[\begin{array}{c} string \\ tension \end{array}\right]$$

self energy = [string tension] $\times$ separation $\Rightarrow$ "confinement"?

Modeling neutron stars with quark cores @ $N_f = 2$





O.Ivanytskyi, D. Blaschke, PRD 105, 114042 (2022)

Agreement with the observational constraints

on mass-radius relation and tidal deformability of neutron stars

Expansion around mean-field solution @ $N_f = 2$

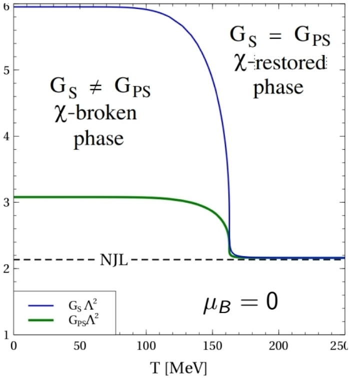
$$\mathcal{U} = \underbrace{\mathcal{U}_{MF}}_{0^{\text{th}}\ \text{order}} + \underbrace{(\bar{q}q - \langle\bar{q}q\rangle)\,\Sigma_{MF}}_{1^{\text{st}}\ \text{order}} - \underbrace{G_S\,(\bar{q}q - \langle\bar{q}q\rangle)^2 - G_{PS}\,(\bar{q}i\vec{\tau}\gamma_5 q)^2}_{2^{\text{nd}}\ \text{order}} + \dots$$

- **Mean-field scalar self-energy**

$$\Sigma_S = \frac{\partial\mathcal{U}_{MF}}{\partial\langle\bar{q}q\rangle}$$

- **Effective medium dependent couplings**

$$G_S = -\frac{1}{2}\frac{\partial^2\mathcal{U}_{MF}}{\partial\langle\bar{q}q\rangle^2}, \qquad G_{PS} = -\frac{1}{6}\frac{\partial^2\mathcal{U}_{MF}}{\partial\langle\bar{q}i\vec{\tau}\gamma_5 q\rangle^2}$$



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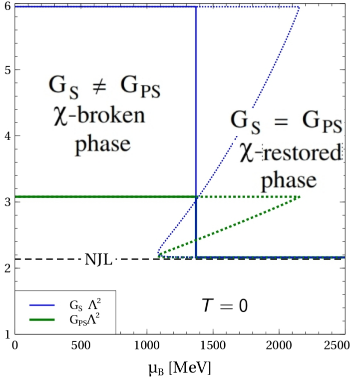


Comparison to NJL model @ $N_f = 2$

$$\mathcal{L} = \bar{q}(i\partial\!\!\!/ - \underbrace{(m + \Sigma_S)}_{\text{effective mass } m^*})q + G_S(\bar{q}q)^2 + G_{PS}\left(\bar{q}i\vec{\tau}\gamma_5 q\right)^2 + \cdots + \mathcal{L}_V + \mathcal{L}_D$$

- **Similarities:**
  - current-current interaction
  - (pseudo)scalar, vector, diquark, ... channels
- **Differences:**
  - high m^* at low T, $\mu \Rightarrow$ **"confinement"**

$$\langle\bar{q}q\rangle = \langle\bar{q}q\rangle_0 \;\Rightarrow\; m^* = m - \frac{2G_0}{3\alpha^{2/3}\langle\bar{q}q\rangle_0^{1/3}}$$

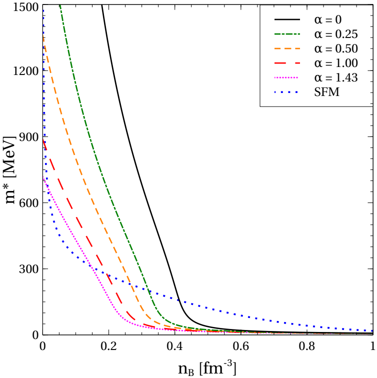
$$\Downarrow$$

$$m^* \to \infty \; at \; \alpha \to 0$$

  - medium dependent couplings:

$$\text{low } T,\ \mu,\ \Rightarrow\ G_S \neq G_{PS}\ \Rightarrow\ \chi\text{-broken}$$
$$\text{high } T,\ \mu,\ \Rightarrow\ G_S = G_{PS}\ \Rightarrow\ \chi\text{-symmetric}$$

$T = 0$



Confining density functional with Polyakov loop @ $N_f = 3$

$$\mathcal{L} = \overline{q}(i\partial\!\!\!/ + gA\!\!\!/ - \hat{m})q - \mathcal{U}_\chi - \mathcal{U}_\Phi, \quad \hat{m} = diag(m_u, m_d, m_s)$$

A_μ - homogeneous static gluon field in the Polyakov gauge

- **Density functional**

$$\mathcal{U}_\chi = D_0 \left[(1+\alpha)\langle\hat{\mathcal{O}}\rangle_0 - \hat{\mathcal{O}}\right]^{1/3}, \quad \hat{\mathcal{O}} = \frac{1}{2}\sum_{a=0,8}\left[(\overline{q}\tau_a q)^2 + (\overline{q}i\gamma_5\tau_a q)^2\right]$$

D. Blaschke, O. Ivanytskyi, M. Shahrbaf, 2202.05061 [nucl-th]

- **Polyakov loop potential**

$$\Phi = \frac{1}{N_c} Tr_c \exp(i\beta A_0), \quad M_H = 1 - 6\bar{\Phi}\Phi + 4\left(\Phi^3 + \bar{\Phi}^3\right) - 3\left(\bar{\Phi}\Phi\right)^2$$

$$\frac{\mathcal{U}_\Phi}{T^4} = -\frac{1}{2}a\bar{\Phi}\Phi + b\log M_H + \frac{1}{2}c\left(\Phi^3 + \bar{\Phi}^3\right) + d\left(\bar{\Phi}\Phi\right)^2$$

T-dependence of a, b, c, d is fitted to the pure SU(3) gauge lattice data

P. M. Lo, B. Friman, O. Kaczmarek, K. Redlich, C. Sasaki, Phys. Rev. D 88, 074502 (2013)

Expansion around mean-field solution @ $N_f = 3$

$$\mathcal{U}_\chi = \underbrace{\mathcal{U}_\chi^{MF}}_{0^{\text{th}}\ \text{order}} + \underbrace{\overline{q}\hat{\Sigma}q - \langle\overline{q}\hat{\Sigma}q\rangle}_{1^{\text{st}}\ \text{order}}$$

$$\underbrace{-\sum_{f,f'}\left(\overline{f}f - \langle\overline{f}f\rangle\right) G_S^{ff'}\left(\overline{f'}f' - \langle\overline{f'}f'\rangle\right) - G_{PS}\sum_f\left(\overline{f}i\gamma_5 f\right)^2}_{2^{\text{nd}}\ \text{order}} + \ldots$$

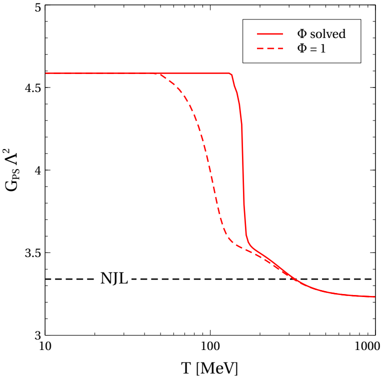
- **Mean-field scalar self-energy**

$$\hat{\Sigma} = diag(\Sigma_u, \Sigma_d, \Sigma_s), \quad \Sigma_f = \frac{\partial\mathcal{U}_\chi^{MF}}{\partial\langle\overline{f}f\rangle}$$

- **Effective medium dependent couplings**

$$G_S^{ff'} = -\frac{1}{2}\frac{\partial^2\mathcal{U}_\chi^{MF}}{\partial\langle\overline{f}f\rangle\partial\langle\overline{f'}f\rangle}$$

$$G_{PS} = -\frac{1}{2}\frac{\partial^2\mathcal{U}_\chi^{MF}}{\partial\langle\overline{f}i\gamma_5 f\rangle^2}$$

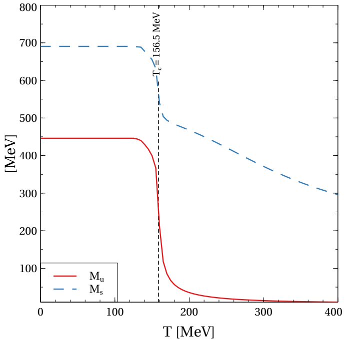


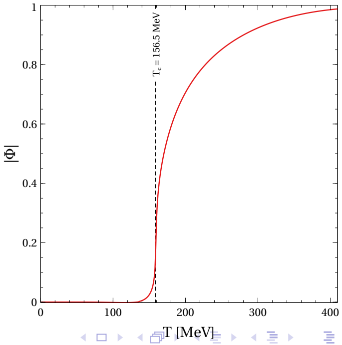
RDF setup

- **Fitting vacuum phenomenology**

$$\begin{cases} M_\pi = 140\ MeV \\ F_\pi = 93\ MeV \\ M_K = 494\ MeV \\ F_K = 112\ MeV \\ T_c = 156.5\ MeV \end{cases} \Rightarrow \quad \begin{array}{l} m_u = m_d = 4.4\ MeV \\ m_s = 134.8\ MeV \\ \Lambda = 636.1\ MeV \\ \sqrt{D_0} = 729.6\ MeV \\ \alpha = 1.44 \end{array}$$

- **Effective masses and Polyakov loop**





Mass-spectrum

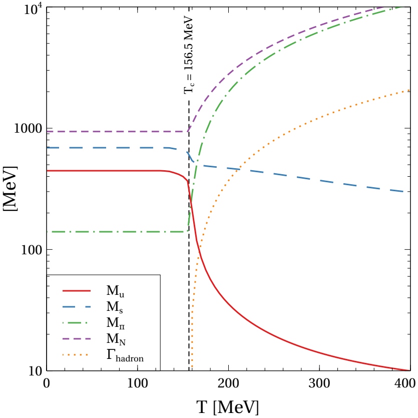
$$M_{n>1} = M_{n>1}^{vacuum} + A(T - T_c)\theta(T - T_c)$$

$$\Gamma_{n>1} = B\theta(T - T_c)$$

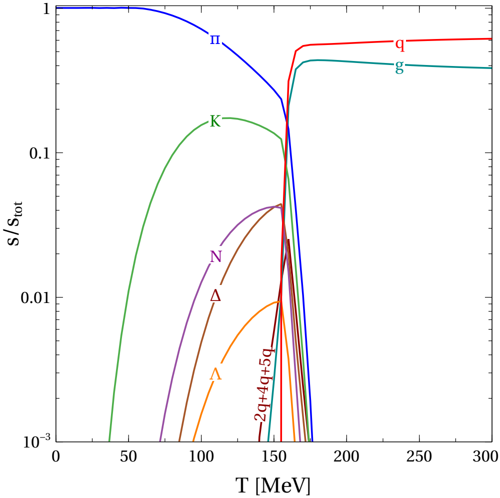
$T_c = 156.6$ MeV, A, B - fitted to lQCD

- **Low T (χ-broken matter)**
  1. heavy quarks
  2. stable multiquark clusters
  3. constant mass of multiquark states
  4. zero width of multiquark states

- **High T (χ-symmetric matter)**
  1. light quarks
  2. unstable multiquark clusters
  3. growing mass of multiquark states
  4. growing width of multiquark states



Composition



Sharp switching between partonic & hadronic degrees of freedom