Eta condensates on the extended Columbia plot with axial anomaly

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Phys. Rev. D 111, 016014

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From the chiral symmetry

$$SU(3)_L \times SU(3)_R \times U(1)_V \times U(1)_A$$

the $U(1)_A$ symmetry is anomalously broken

Physical consequences:
e.g. mass of η and η'

Phys.Rept. 142 (1986) 357-387

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around the transition temperature
(but above or below?)

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Phys.Rept. 142 (1986) 357-387

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Axial anomaly and the order of the phase transition in the chiral limit:

From ϵ expansion **Phys. Rev. D 29 (1984) 338**

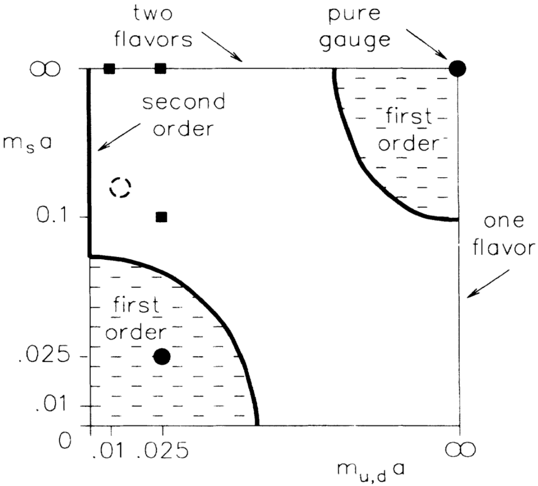
In the chiral limit:

- $N_f = 0$ Pure Yang-Mills, first-order.
- $N_f = 1$ Crossover transition with the anomaly. (can be second order without)
- $N_f = 2$ Without anomaly the transition is second order, with anomaly it might be first or second order depending on the restoration of the anomaly.
- $N_f = 3$ First-order transition.

The chiral symmetry is explicitly broken by $m_u \approx m_d > 0$ and $m_s > 0$

Columbia plot:
the order of the phase transition in the parameter space of
$m_{u,d}, m_s \; || \; h_N, h_S \; || \; m_K, m_\pi$

Phys Rev Lett 65 (1990) 2491



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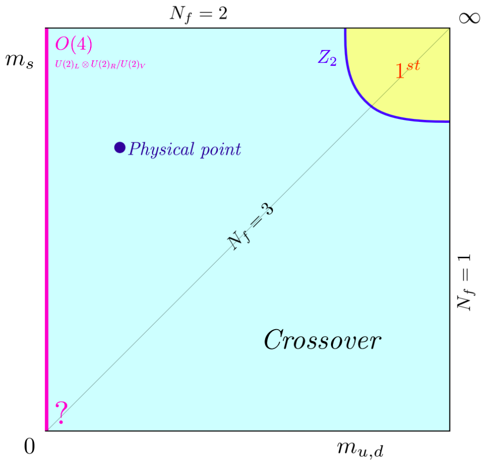
Phys Rev Lett 65 (1990) 2491

No first-order in bottom left?
Symmetry 13 (2021) 11, 2079
Phys.Rev.D 108 (2023) 11, 114018

Order in top left?
Phys.Rev.D 110 (2024) 1, 016021



Higher order terms that can break the $U(1)_A$ symmetry
Terms allowed by symmetry considerations (breaking $U(1)_A$, but not other)

$$\xi_1\left(\det\Phi + \det\Phi^\dagger\right),\quad \xi_1^1\,\mathrm{tr}\left(\Phi^\dagger\Phi\right)\left(\det\Phi + \det\Phi^\dagger\right),\quad \xi_2\left[\left(\det\Phi\right)^2 + \left(\det\Phi^\dagger\right)^2\right],\quad \xi_2^\pm\left(\det\Phi \pm \det\Phi^\dagger\right)^2$$

- higher order $|Q| > 2$
- $\log\det$ term á la Witten, di Vecchia, Veneciano
- polydeterminant **Lett. Math. Phys. 115, no.6, 119 (2025)**

Model at hand:
Extended Linear Sigma Model (eLSM)

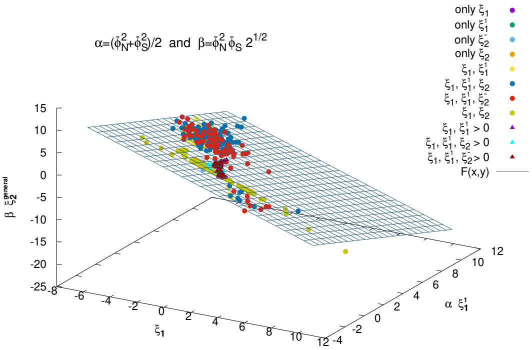
Phys. Rev. D 87, no.1, 014011 (2013)
Phys. Rev. D 93, no.11, 114014 (2016)
Phys. Rev. D 104, 056013 (2021)

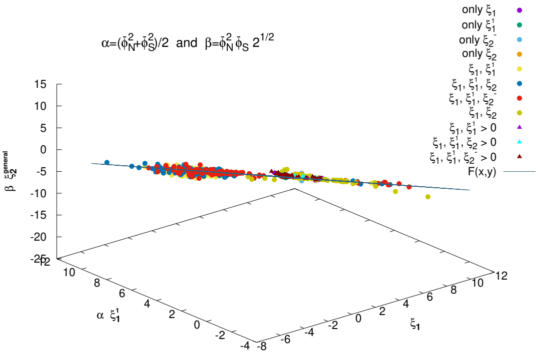
Only an effective coupling matters

Phys.Rev.Lett. 132 (2024) 25, 251903

Comparing m_π^2 and m_η^2: $\xi_{\text{eff}} \equiv \xi_1 + \xi_1^1\left(\phi_N^2 + \phi_S^2\right)/2 + \xi_2\phi_N^2\phi_S/\sqrt{2}$

Phys. Rev. D 111 (2025), 016014





At finite temerature $\xi_{\text{eff}} = \xi_{\text{eff}}(T)$ due to $\phi_N(T)$ and $\phi_S(T)$

Keep only ξ_1 and ξ_2 and compare to $\xi_{\rm eff} = 1.50$ GeV. Using h_N and h_S (alternative: m_π and m_K)

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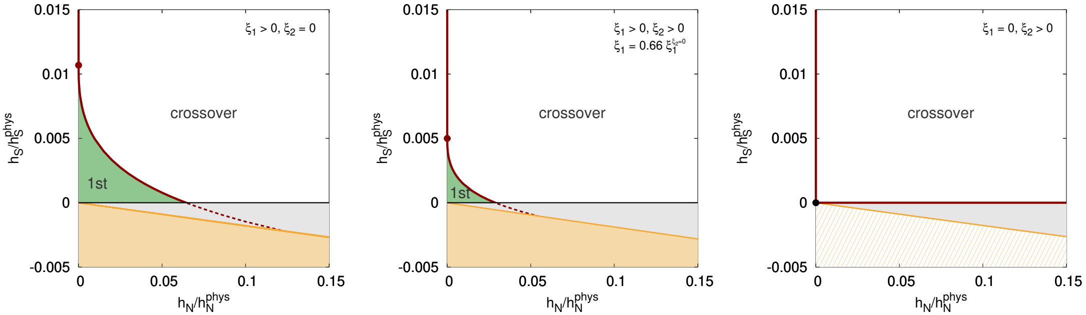

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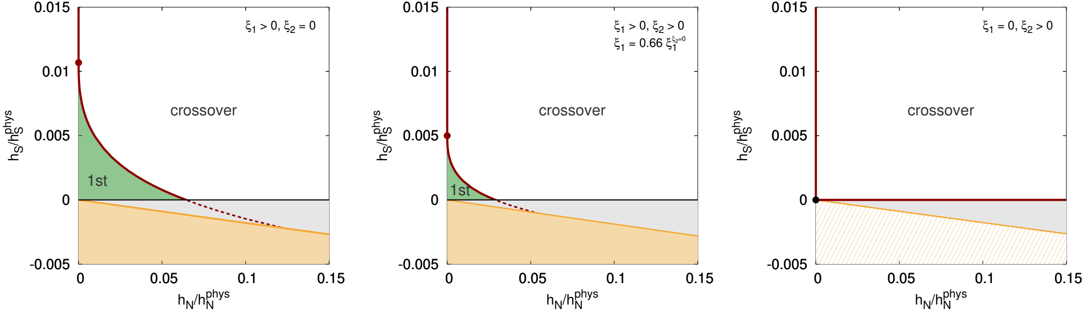

Size of first-order region in controlled by ξ_1.

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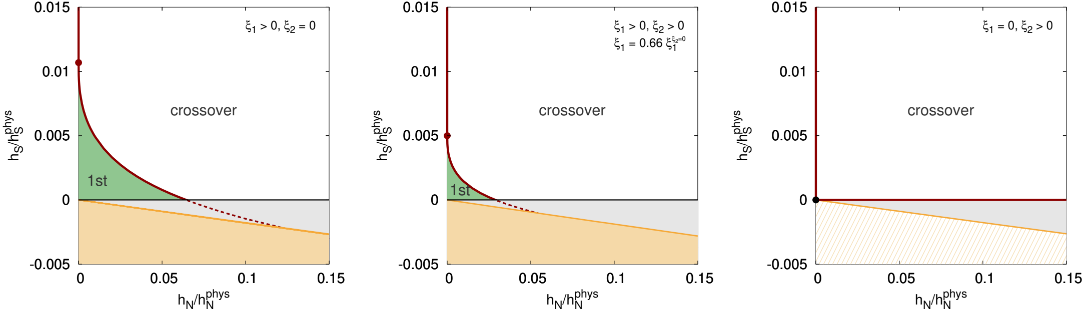

Size of first-order region in controlled by ξ_1.

At $h_S < 0$ (already at $m_K > 0$): eta condensation $\Rightarrow$ **CP violation** depending on the anomaly terms!

Topological angle:

QCD vacuum: topological sectors with different topological charge

Topological angle $\theta > 0$: CP violation; first-order transition at $\theta = \pi$

Effective model: θ appears in the anomaly term

Phys. Rev. Res. 2, no.3, 033359 (2020)

$$\xi_1 \left(e^{i\theta} \det \Phi + e^{-i\theta} \det \Phi^\dagger \right)$$
$$\xi_2 \left[e^{i2\theta} \left(\det \Phi \right)^2 + e^{-i2\theta} \left(\det \Phi^\dagger \right)^2 \right]$$

$$\xLeftrightarrow[\alpha=\theta]{s \to e^{i\frac{\alpha}{2}\gamma_5} s}$$

$$\begin{aligned} h_s \phi_s &\to h_s \cos\theta \, \phi_s + h_s \sin\theta \, \eta_s \\ &= h_s^{\mathrm{scal}} \, \phi_s + h_s^{\mathrm{pseu}} \, \eta_s \end{aligned}$$

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For $\theta = \pi \qquad h_s \phi_s \to -h_s \phi_s$

negative explicit breaking $\sim$ negative m_s

(disfavored only by the anomaly!)

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Daschen's phenomena $\Leftarrow$

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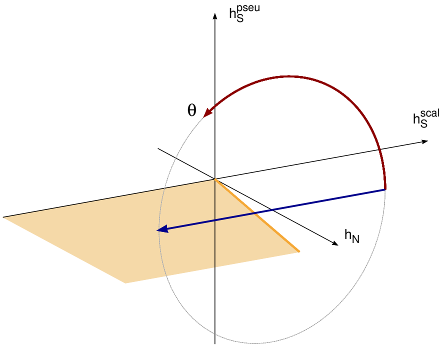
Varying h and θ $\Leftrightarrow$ **Varying h^{scal} and h^{pseu}**



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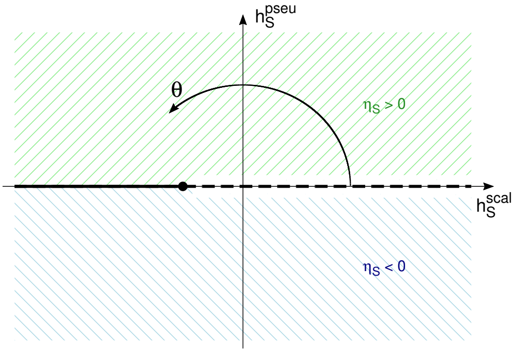

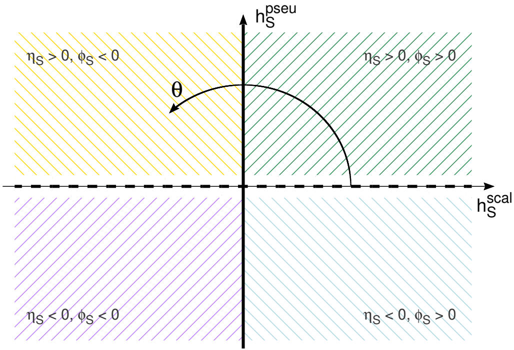

With only the 't Hooft term

Only higher order term

- Multiple terms to describe the $U(1)_A$ anomaly (almost) equally well, only an effective coupling can be determined.
- The relative size of 't Hooft term determines the size of the first-order region in the bottom left corner of the Columbia pot.
- Eta condensates emerge at negative explicit breaking $\Leftrightarrow$ at $\theta = \pi$ depending in the anomaly terms.

- $h_s < 0$ 2+1 flavor calculation tell about the content of ξ_{eff}.
- Coumbia plot and CP violation depends on the anomaly terms.
- Higher order terms ($\det^2$, $\det^3$, general) and their effect is interesting.

Thank You!