Quantum mechanics in heavy-ion collisions

Outline

- Classical (non-quantum) models at scales much shorter than the double slit experiment?
- Large quantum corrections to kinetic theory, hydro results still ok
- Only some classical results still valid

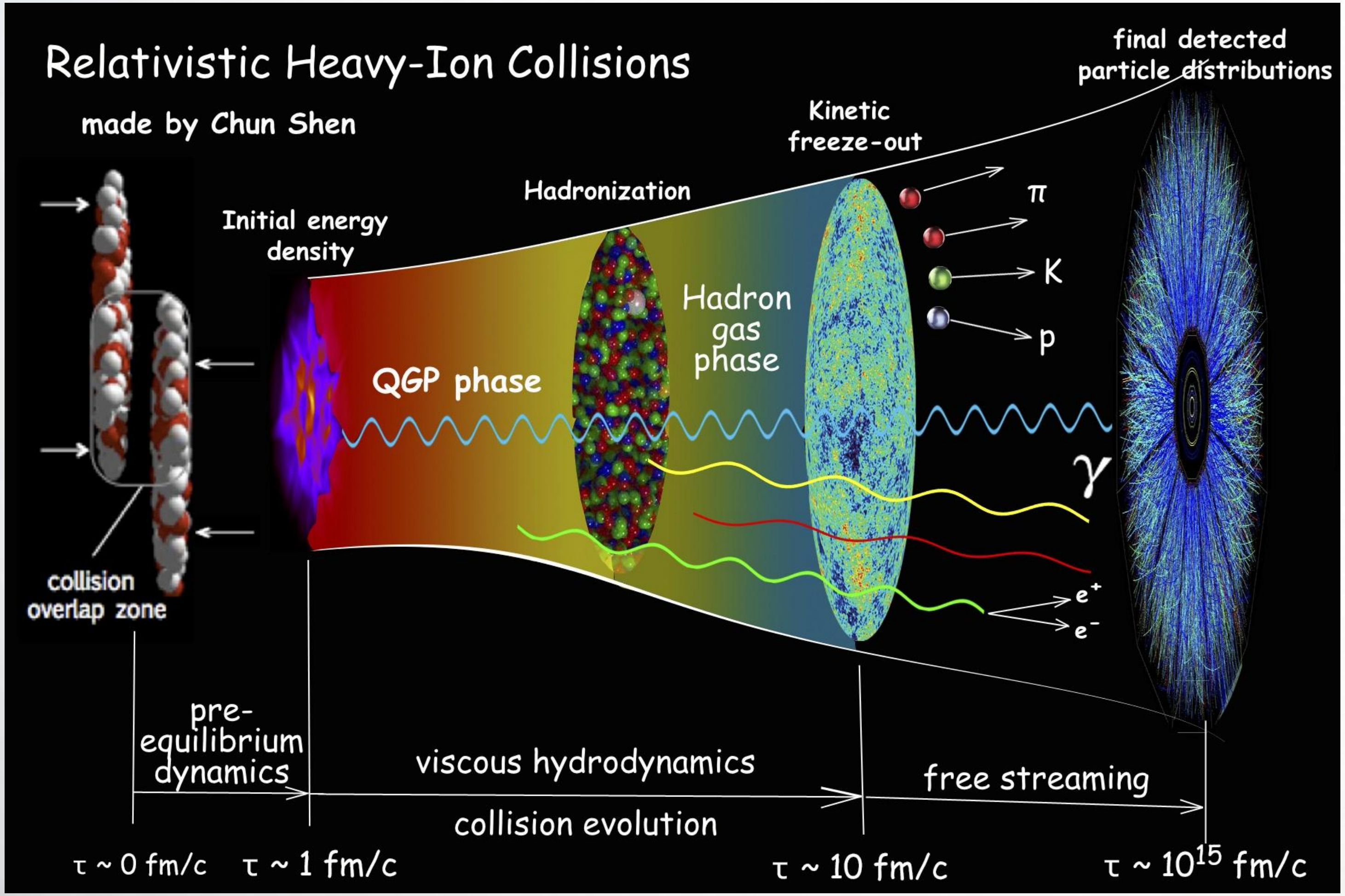
Uniwersytet Jana Kochanowskiego w Kielcach

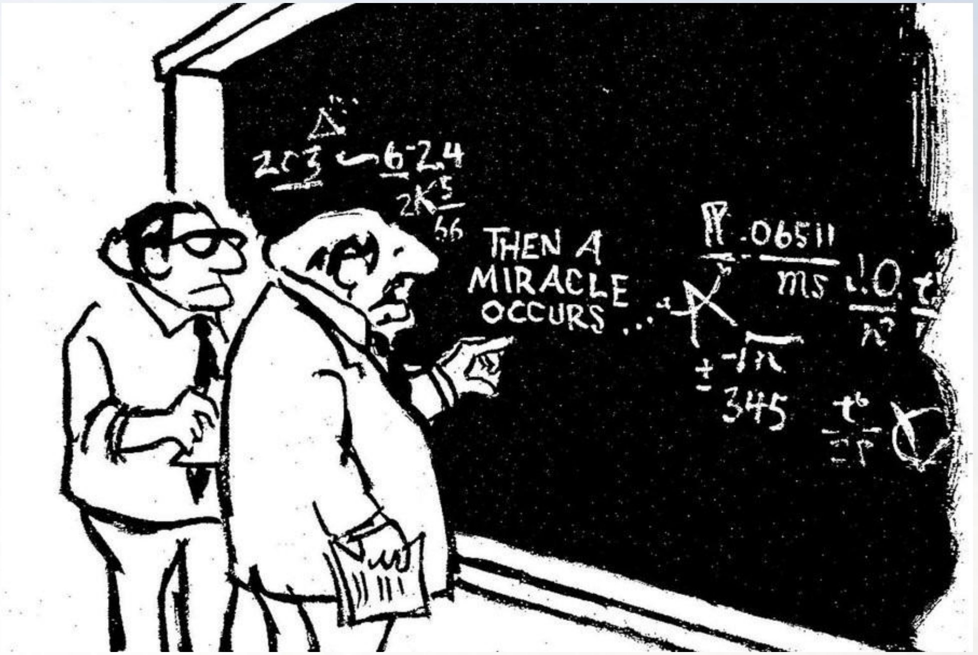
Leonardo Tinti, 14-12-2025,

XVIII Polish Workshop on Relativistic Heavy-Ion Collisions: Strange and Heavy Flavour Physics

Magic box

First, global equilibrium (not as silly as it seems)





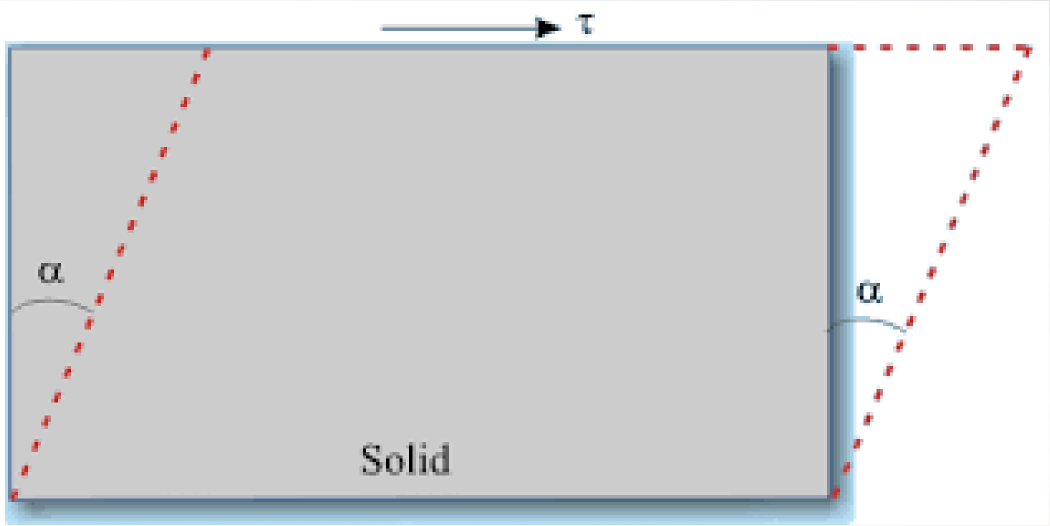
$$\hat{\rho} = \sum_i P_i \, |\psi_i\rangle\langle\psi_i|$$

Next (perfect) **fluid, then particles**
(viscous, then kinetic etc.)

Some definitions from classical mechanics

$$\partial_t \rho + \nabla_{\mathbf{x}} \cdot (\rho\, \boldsymbol{v}) = 0$$

$$\rho \boldsymbol{a} = -\nabla_{\mathbf{x}} \mathrm{P}$$



generalizes to

$$\rho \boldsymbol{a} = \nabla_{\boldsymbol{x}} \cdot T$$

the Cauchy Tensor T

definition of a fluid:

$$T_{ij}\Big|_{eq} = -\mathrm{P}\, \delta_{ij}$$

For an incompressible fluid

$$\nabla_{\boldsymbol{x}} \cdot \boldsymbol{v} = \mathbf{0}$$

$$T_{ij} \simeq -\mathrm{P}\, \delta_{ij} + \eta \left(\partial_i v_j + \partial_j v_i\right) + \cdots$$

Then, in special relativity

$$\partial_\mu T^{\mu\nu} = 0$$

$$T^{\mu\nu} = \mathcal{E}\, u^\mu u^\nu - \mathcal{P} \Delta^{\mu\nu} + \delta T^{\mu\nu}$$

Relativistic kinetic theory would be a very convenient background (hydro-wise)

Relativistic Boltzmann equation

$$p \cdot \partial f = -\mathcal{C}[f]$$

$$\Rightarrow \int_{\boldsymbol{p}} p^{\nu} p \cdot \partial f = -\int_{\boldsymbol{p}} p^{\nu} C = 0$$

$$\partial_{\mu} T^{\mu\nu}$$

$$p \cdot \partial = (p \cdot u)(u \cdot \partial) + p \cdot \nabla$$

$$u \cdot \partial f = \dot{f} = -\frac{p \cdot \nabla f}{(p \cdot u)} - \frac{C[f]}{(p \cdot u)}$$

extra needed equations

$$\dot{T}^{\mu\nu} = \int_{\boldsymbol{p}} p^{\mu} p^{\nu} \dot{f}$$

covariant momentum integral

$$\int_{\mathbf{p}} = \int d^4p\, 2\Theta(p_0)\delta(p^2 - m^2)$$

G Denicol, J.Phys. G41 (2014) no.12, 124004

Relativistic kinetic theory would be a very convenient background (hydro-wise)

$\mathcal{O}^{\langle\mu_1\rangle\cdots\langle\mu_l\rangle} = \Delta^{\mu_1}_{\alpha_1}\cdots\Delta^{\mu_l}_{\alpha_l}\mathcal{O}^{\alpha_1\cdots\alpha_l}$ even more convenient basis $\mathfrak{f}_r^{\mu_1\cdots\mu_l} = \int_{\boldsymbol{p}} (p\cdot u)^r\, p^{\langle\mu_1\rangle\cdots\langle\mu_l\rangle} f$

$$\partial_\mu u_\nu = u_\mu \dot{u}_\nu + \sigma_{\mu\nu} + \omega_{\mu\nu} + \frac{1}{3}\theta\Delta_{\mu\nu}, \qquad T^{\mu\nu} = \mathcal{E}u^\mu u^\nu + \mathcal{P}^{\mu\nu} = \mathcal{E}u^\mu u^\nu - (\mathcal{P}+\Pi)\Delta^{\mu\nu} + \pi^{\mu\nu}$$

$$\begin{cases} u_\nu \partial_\mu T^{\mu\nu} = 0 \\ \partial_\mu T^{\mu\langle\nu\rangle} = 0 \end{cases} \Rightarrow \begin{cases} \dot{\mathcal{E}} = -\theta(\mathcal{E}+\mathcal{P}+\Pi) + \pi^{\mu\nu}\sigma_{\mu\nu} \\ (\mathcal{E}+\mathcal{P}+\Pi)\,\dot{u}^\nu = \nabla^\nu(\mathcal{P}+\Pi) - \nabla_\mu \pi^{\mu\langle\nu\rangle} + \pi^{\nu\alpha}\dot{u}_\alpha \end{cases}$$

$$\mathcal{E} = \mathfrak{f}_2 \qquad \mathcal{P}^{\mu\nu} = -(\mathcal{P}+\Pi)\Delta^{\mu\nu} + \pi^{\mu\nu} = \mathfrak{f}_0^{\mu\nu}$$

Method of moments (and its quantum generalization?)

$$u \cdot \partial f = \dot{f} = -\frac{p \cdot \nabla f}{(p \cdot u)} - \frac{C[f]}{(p \cdot u)}$$

$$\dot{T}^{\mu\nu} = \int_{\boldsymbol{p}} p^{\mu} p^{\nu} \dot{f}$$

in particular

$$\dot{\mathcal{P}}^{\langle\mu\rangle\langle\nu\rangle} + C_{-1}^{\langle\mu\rangle\langle\nu\rangle} = 2(\mathcal{P} + \Pi)\sigma^{\mu\nu} + \frac{5}{3}\theta(\mathcal{P} + \Pi)\Delta^{\mu\nu} - \frac{5}{3}\theta\pi^{\mu\nu} - 2\pi_{\alpha}^{(\mu}\sigma^{\nu)\alpha} + 2\pi_{\alpha}^{(\mu}\omega^{\nu)\alpha}$$
$$-\nabla_{\alpha}\mathfrak{f}_{-1}^{\alpha\langle\mu\rangle\langle\nu\rangle} - \left(\sigma_{\alpha\beta} + \frac{1}{3}\theta\Delta_{\alpha\beta}\right)\mathfrak{f}_{-2}^{\alpha\beta\mu\nu}$$

…but no immediate generalization of the higher rank tensors!

$$\mathfrak{f}_{r}^{\mu_1\cdots\mu_l} = \int_{\boldsymbol{p}} (p \cdot u)^r \, p^{\langle\mu_1\rangle\cdots\langle\mu_l\rangle} f$$

Large quantum corrections, compared to the kinetic limit

$$W(x,k) = \frac{2}{(2\pi)^4}\int d^4v\, e^{-ik\cdot v}\ \mathrm{tr}\left(\hat{\rho}\,\widehat{\Phi}^{\dagger}(x+v/2)\widehat{\Phi}(x-v/2)\right)$$

quantum precursor of the distribution function

$$T^{\mu\nu} = \int d^4k\, k^{\mu}k^{\nu}\, W(x,k)$$

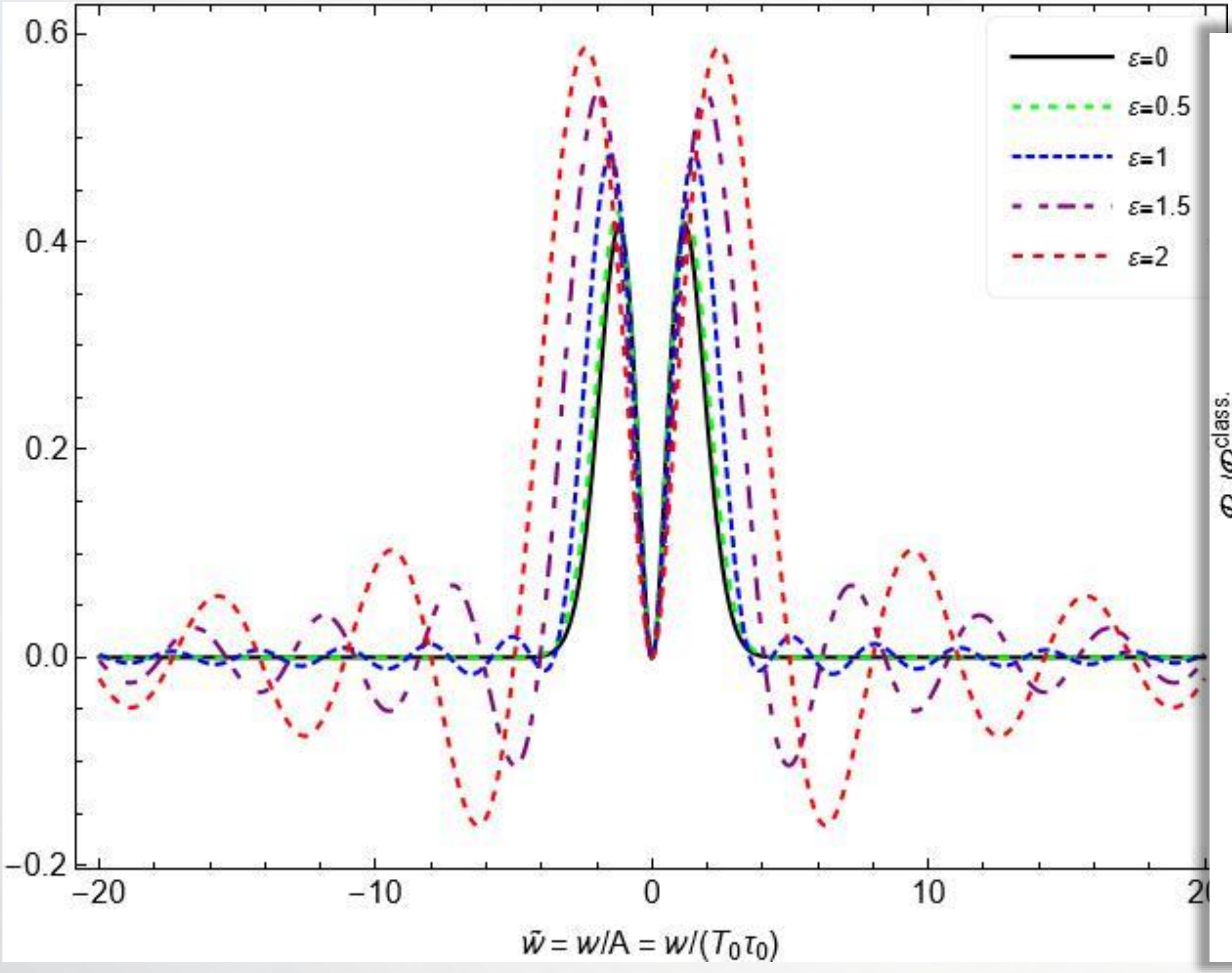
- Relativistic Kinetic Theory. Principles and Applications - De Groot, S.R. et al. Amsterdam, Netherlands: North-Holland (1980)

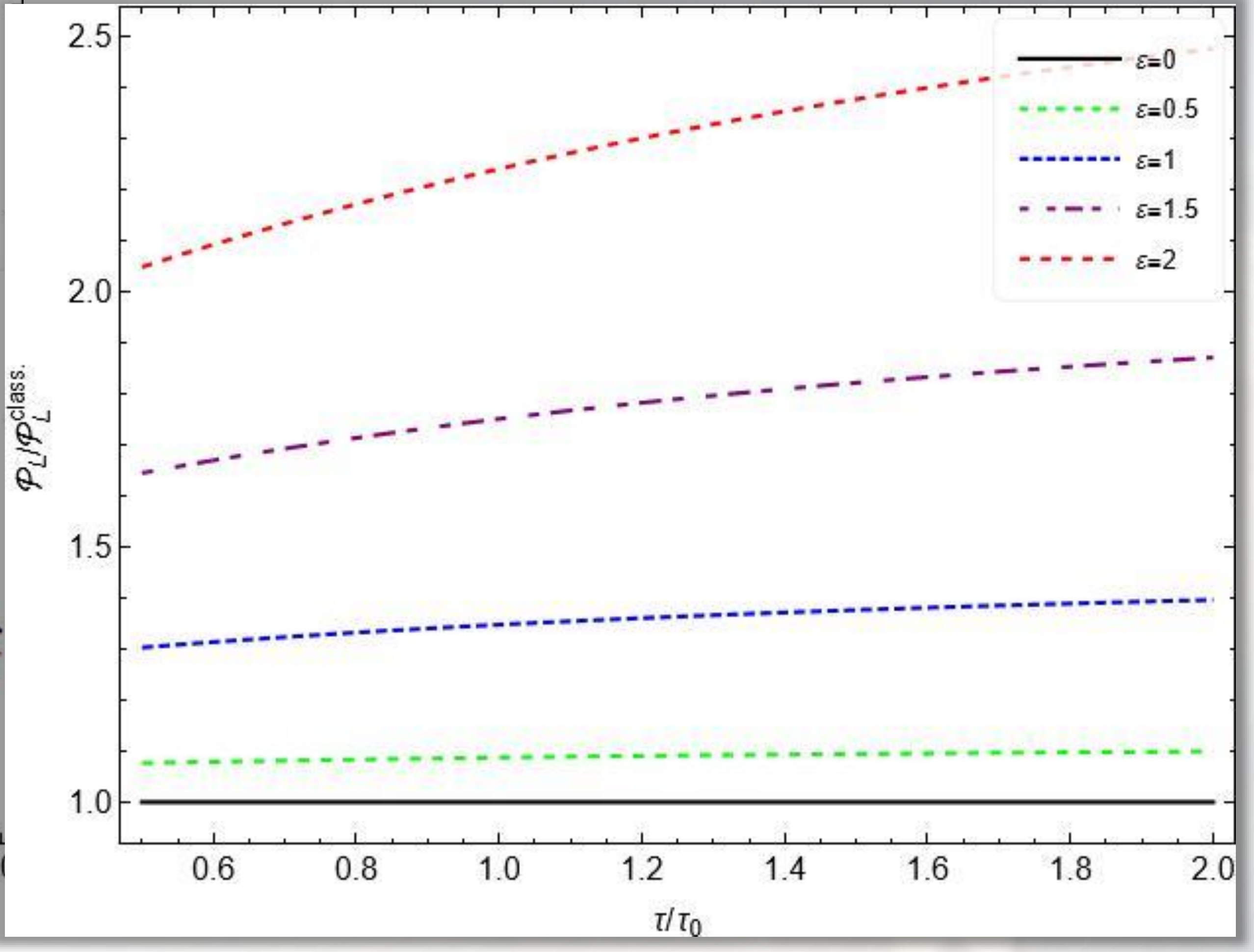
$$\left[\frac{1}{4}\hbar^2\Box - \left(k^2 - m^2c^2\right) + i\hbar k\cdot\partial\right] W(x,k) = \cdots$$

general form of the exact solutions for the (1+1)d expansion

<u>L T</u>, Phys.Rev.D 108 (2023) 7, 076022

Large quantum corrections, compared to the kinetic limit

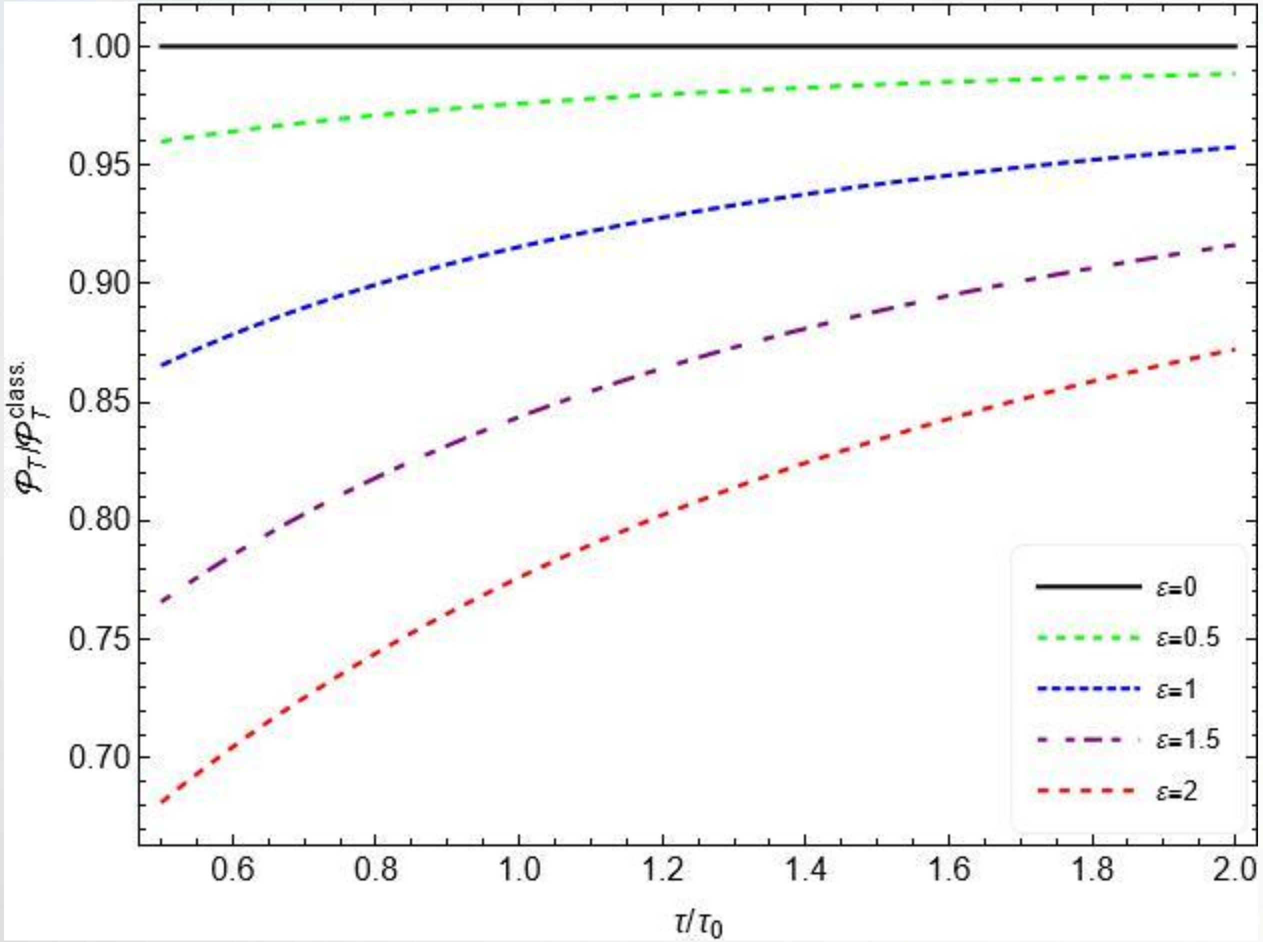


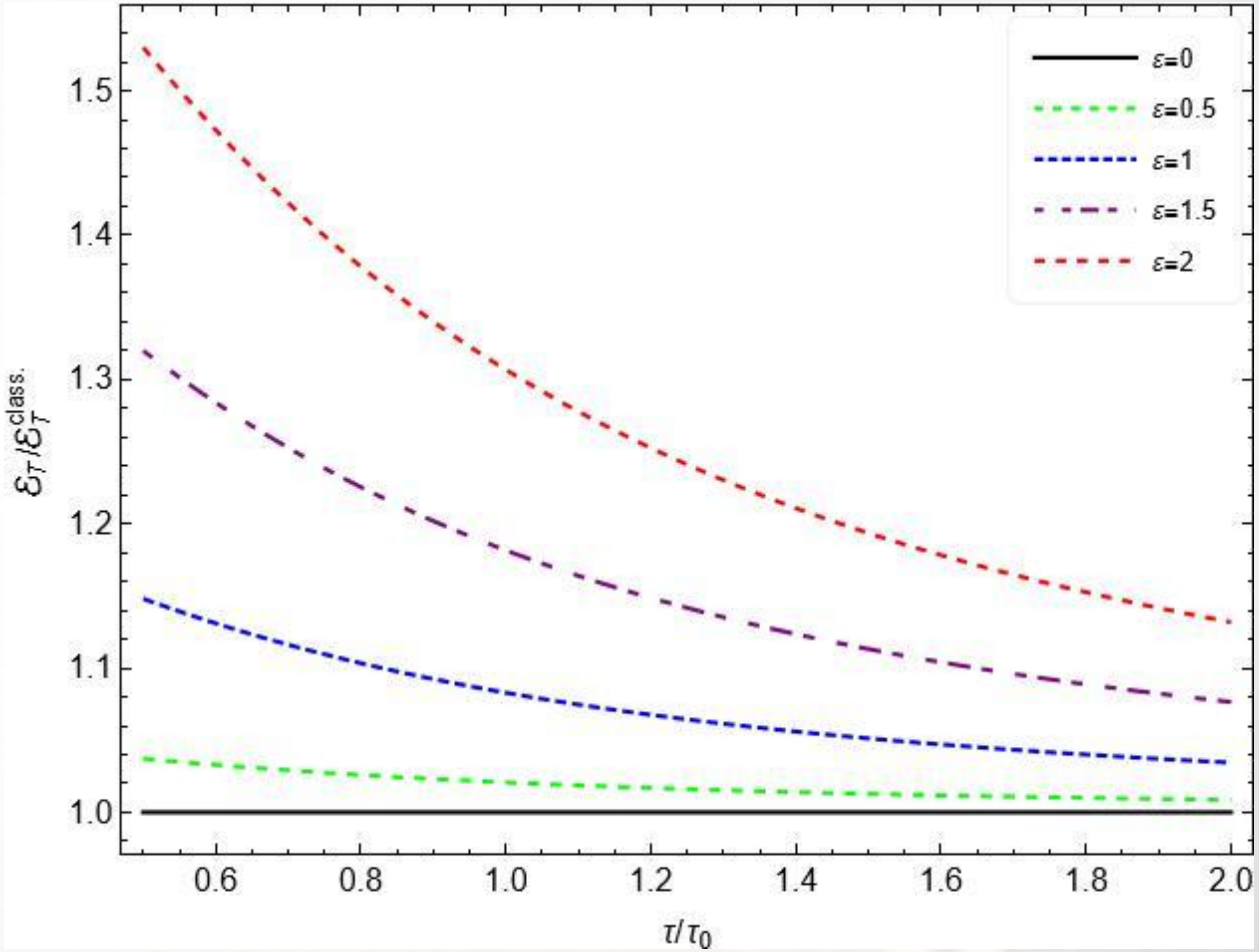


The small fluctuations in the long tails are relevant(even if very close the kinetic values)

L T, Phys.Rev.D 108 (2023) 7, 076022

Large quantum corrections, compared to the kinetic limit





Similar situation for the transverse pressure (the corrections in the opposite direction)

L T, Phys.Rev.D 108 (2023) 7, 076022

Generalization of the method of moments

Solution: add a $(\boldsymbol{k}\cdot\boldsymbol{u})^n e^{-\zeta(\mathrm{k}\cdot\boldsymbol{u})^2}$ to the definition of the moments (to keep the power of $(\boldsymbol{k}\cdot\boldsymbol{u})$ positive)

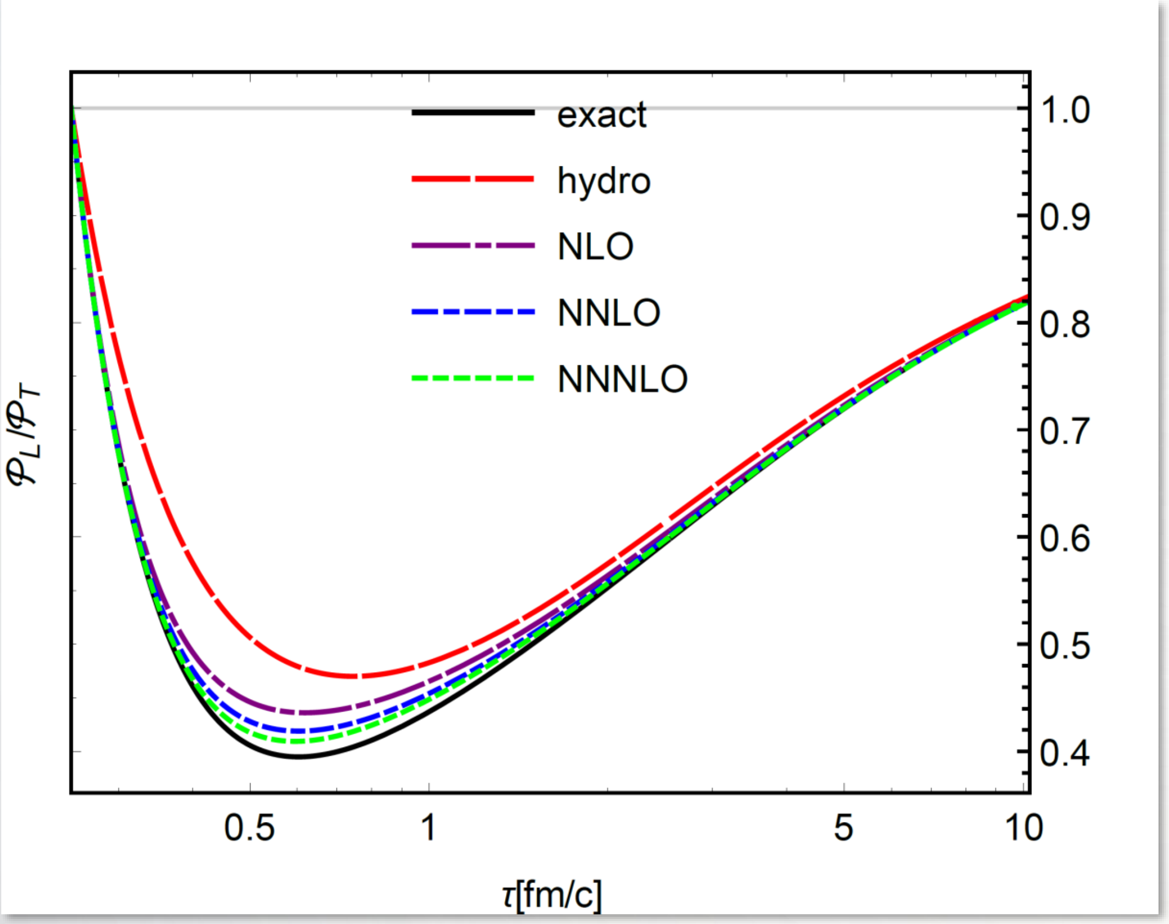
$$\phi_n^{\mu_1\cdots\mu_s}(x,\zeta) = \int \frac{d^4k}{(2\pi)^4}\,(k\cdot u)^n\, e^{-\zeta(k\cdot u)^2} k^{\langle\mu_1\rangle}\ldots k^{\langle\mu_s\rangle} W(x,k)$$

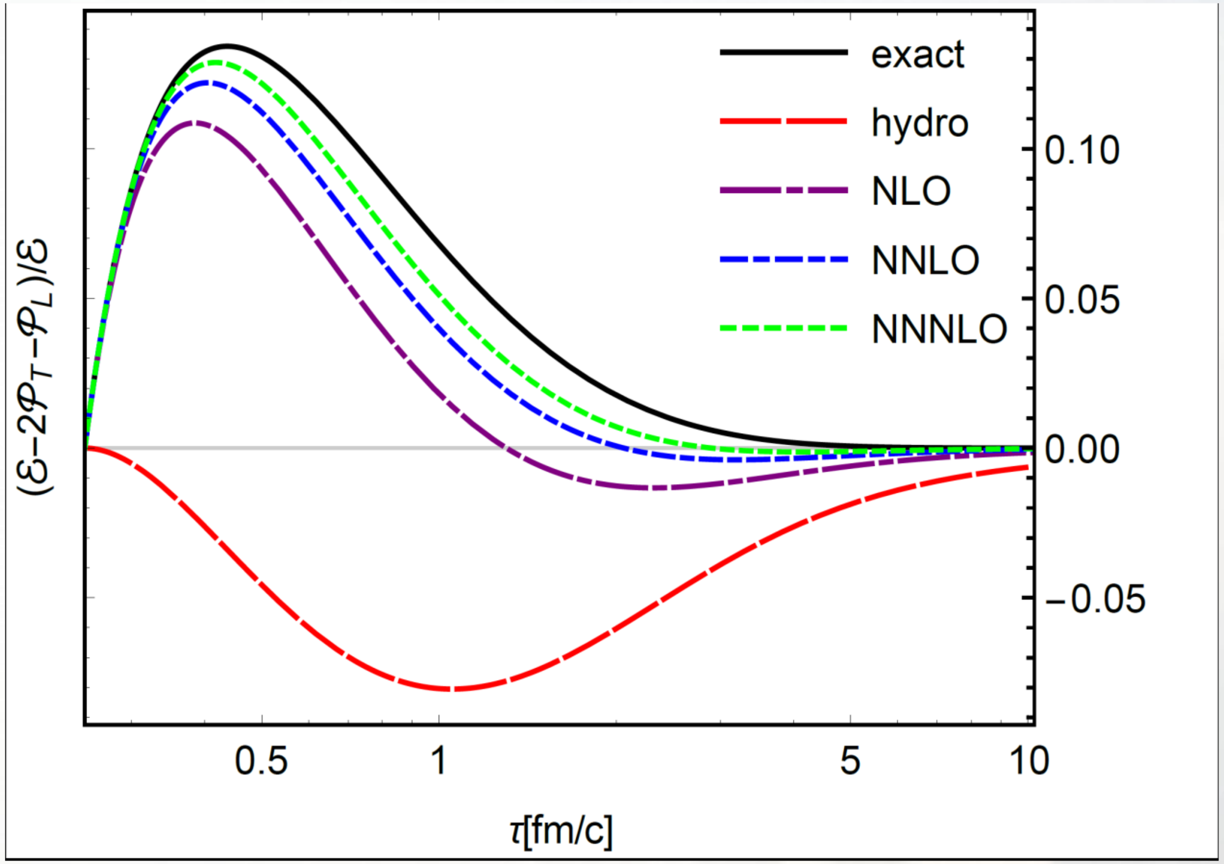
$$\dot{\mathcal{P}}^{\langle\mu\rangle\langle\nu\rangle} = 2(\mathcal{P}+\Pi)\sigma^{\mu\nu} + \frac{5}{3}\theta(\mathcal{P}+\Pi)\Delta^{\mu\nu} - \frac{5}{3}\theta\pi^{\mu\nu} - 2\pi_\alpha^{(\mu}\sigma^{\nu)\alpha} + 2\pi_\alpha^{(\mu}\omega^{\nu)\alpha}$$
$$+\int_0^\infty d\zeta\left\{\tilde{C}_1^{\langle\mu\rangle\langle\nu\rangle} - \nabla_\alpha\phi_1^{\alpha\langle\mu\rangle\langle\nu\rangle} + \dot{u}_\alpha\left[2\phi_1^{\alpha\mu\nu} - 2\zeta\phi_3^{\alpha\mu\nu}\right] + \nabla_\alpha u_\beta\left[\phi_0^{\alpha\beta\mu\nu} - 2\zeta\phi_2^{\alpha\beta\mu\nu}\right]\right\}$$

In simple enough cases, the underlying W(x,k) can be solved as well as the approximations in the generalized hydrodynamic expansion

L T, Phys.Rev.D 108 (2023) 3, 036015

Numerical tests of the generalized hydrodynamic expansion

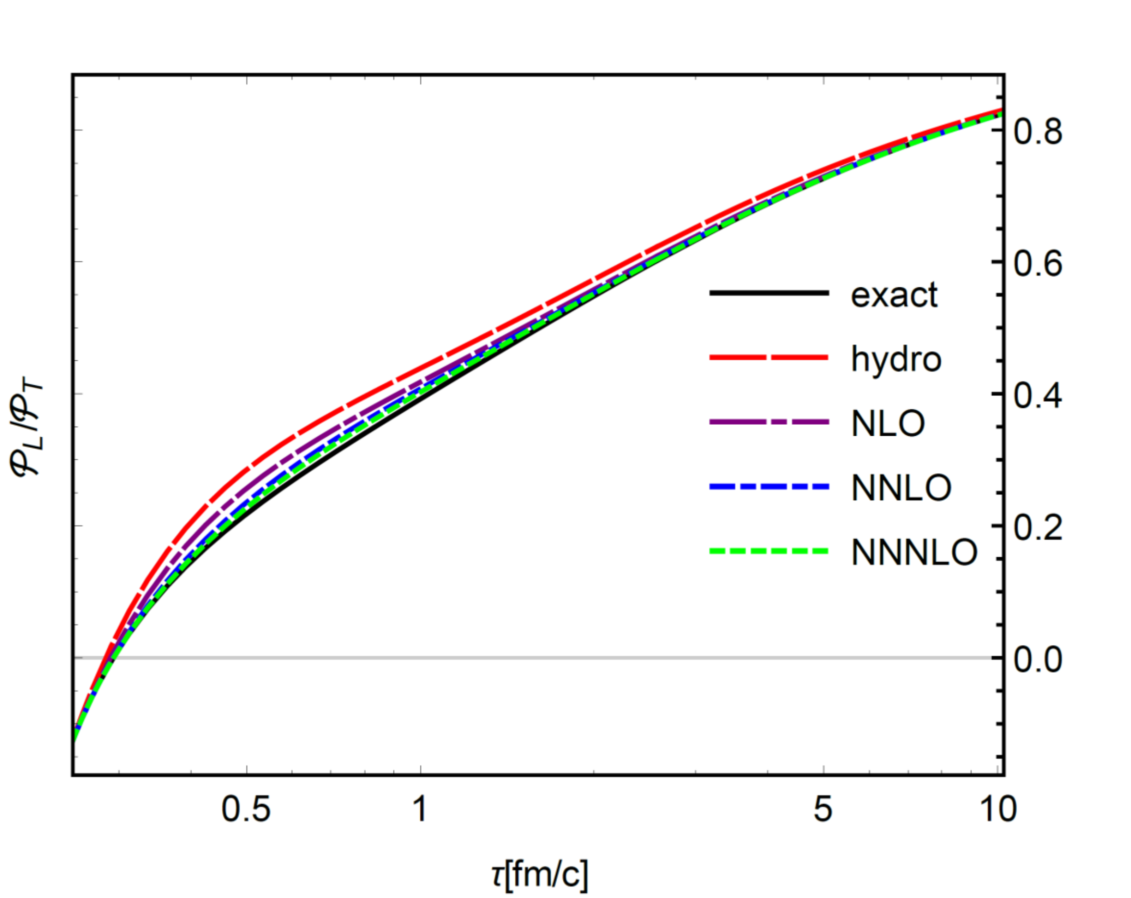


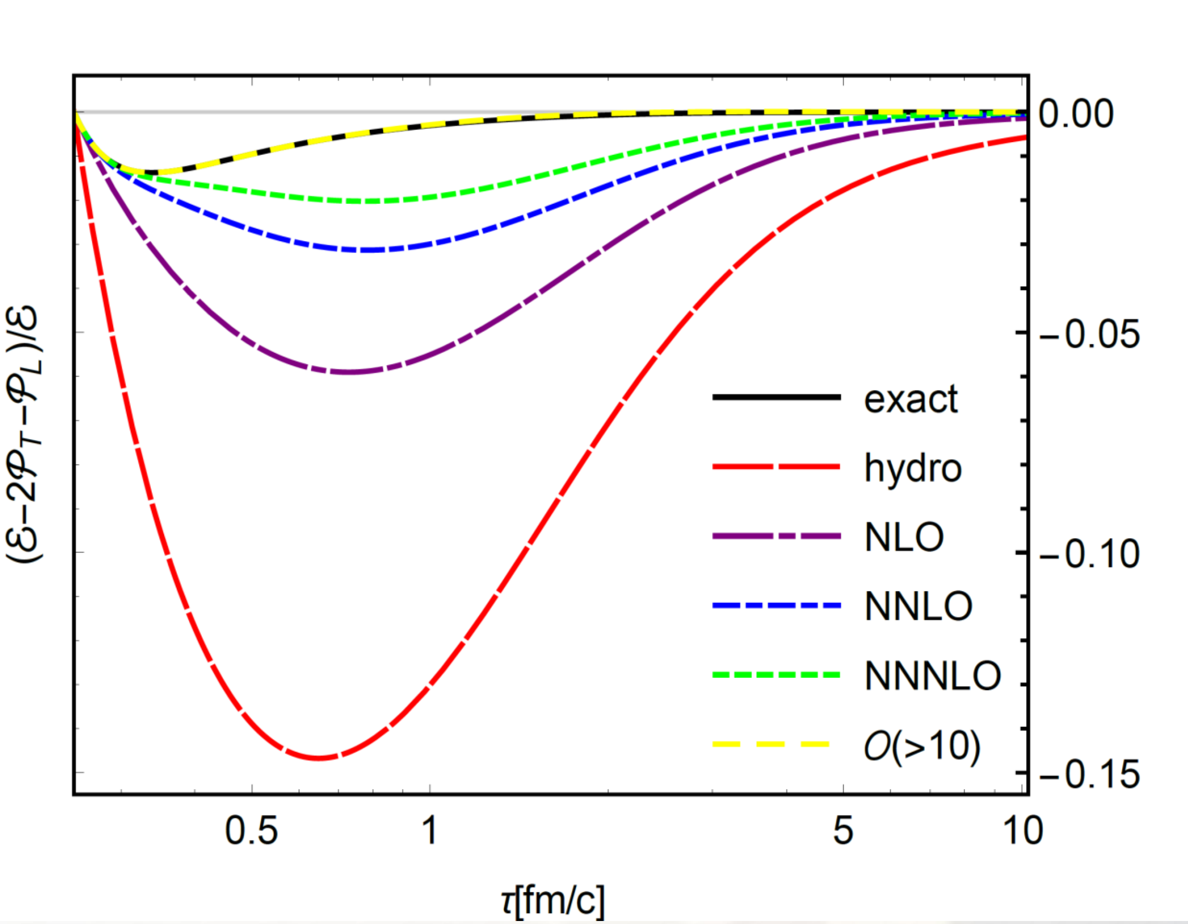


Possibility to estimate if hydro is going to be a good approximation, or not…

L T, Phys.Rev.D 108 (2023) 3, 036015

Numerical tests of the generalized hydrodynamic expansion





Possibility to estimate if hydro is going to be a good approximation, or not…

<u>L T</u>, Phys.Rev.D 108 (2023) 3, 036015

None of this works for other classical expectations

$$W(x,k) = \frac{2}{(2\pi)^4}\int d^4v\, e^{-ik\cdot v}\ \mathrm{tr}\left(\hat{\rho}\,\hat{\Phi}^{\dagger}(x+v/2)\hat{\Phi}(x-v/2)\right)$$

non-vanishing commutator at space-like distances

$W(x,k)$ is not even a candidate for a local observable in quantum field theory!

See, as a reference (warning math heavy book!)

- Local Quantum Physics: Fields, Particles, Algebras – R. Haag. Springer (1996, II edition)

Summary and outlook

- No need to go to the kinetic limit, off-shell hydrodynamic expansion
- Important role of the symmetry
- Other classical effects must be checked, rather than assumed: The success of hydro and transport is not sufficient.

Thank you for your attention!

Back up slides

$$\int [g(x) + h(x)]\, dx \neq \int g(x)dx + \int h(x)dx$$

$$\int \lim_{\varepsilon\to 0} f(\varepsilon, x)dx \neq \lim_{\varepsilon\to 0} \int f(\varepsilon, x)dx$$

$$\frac{1}{\beta} = \int_0^\infty \left[-\partial_\beta \left(\frac{e^{-\beta x}}{x}\right)\right] dx \neq -\partial_\beta \left(\int_0^\infty \frac{e^{-\beta x}}{x} dx \equiv \infty\right)$$

$$\frac{1}{x} = \int_0^\infty e^{-\alpha x} d\alpha$$

$$\frac{1}{(\alpha+\beta)^2} = \int_0^\infty dx \left[-\partial_\beta\left(e^{-(\alpha+\beta)x}\right)\right] = -\partial_\beta \left(\int_0^\infty dx\, e^{-(\alpha+\beta)x} = \frac{1}{\alpha+\beta}\right),$$

$$\int_0^\infty d\alpha \left[\frac{1}{(\alpha+\beta)^2} = \partial_\alpha\left(-\frac{1}{\alpha+\beta}\right)\right] = \frac{1}{\beta}$$

What's wrong with the relativistic kinetic theory?

Probability density ρ in a 6N+1 dimensional space

$$\rho(\boldsymbol{x}_1, \boldsymbol{x}_2, \cdots, \boldsymbol{x}_N; \boldsymbol{p}_1, \boldsymbol{p}_2, \cdots, \boldsymbol{p}_N; t)$$

integration over all particles, but one

$$f(\boldsymbol{x}, \boldsymbol{p}; t) = \sum_i \int \prod_{j \neq i} \frac{d^3 x_j d^3 p_j}{(2\pi\hbar)^3} \, \rho$$

normalized to N (by construction)

$$N = \int \frac{d^3 x d^3 p}{(2\pi\hbar)^3} f(\boldsymbol{x}, \boldsymbol{p}; t)$$

The distribution function $f(\boldsymbol{x}, \boldsymbol{p}; t)$

Then molecular chaos, and further approximations

What's wrong with the relativistic kinetic theory?

The relativistic Boltzmann equation

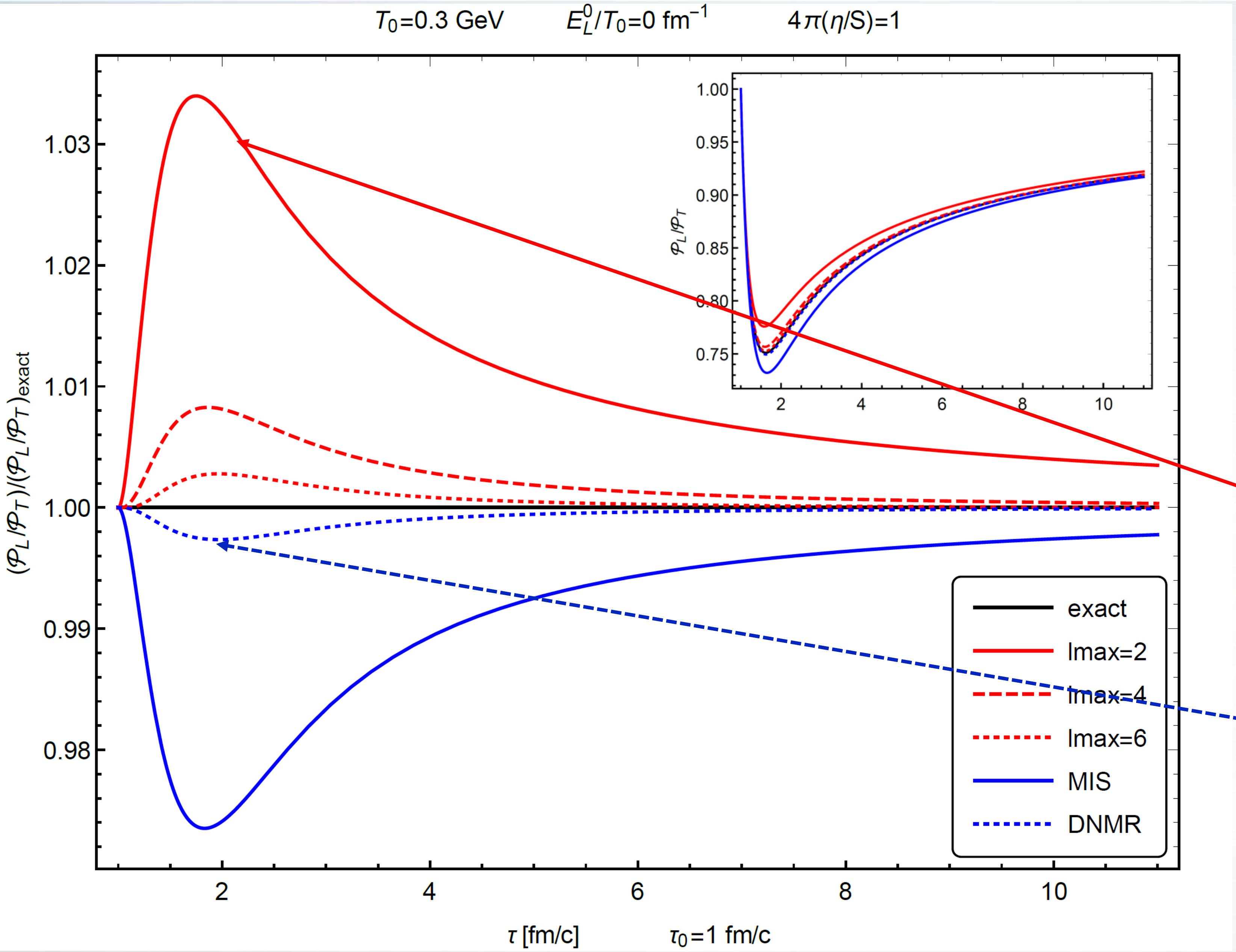
$$p \cdot \partial f(x, \mathbf{p}) = -C[f, \bar{f}]$$
$$p \cdot \partial \bar{f}(x, \mathbf{p}) = -\bar{C}[f, \bar{f}]$$

Kinetic only contributions to $T^{\mu\nu}$ and to J^{μ}

$$T^{\mu\nu}(x) = \frac{g_S}{(2\pi\hbar)^3} \int \frac{d^3p}{E_{\mathbf{p}}} \, p^{\mu} p^{\nu} \left(f(x, \mathbf{p}) + \bar{f}(x, \mathbf{p}) \right)$$

$$J_B^{\mu}(x) = \frac{g_S}{(2\pi\hbar)^3} \int \frac{d^3p}{E_{\mathbf{p}}} \, p^{\mu} \left(f(x, \mathbf{p}) - \bar{f}(x, \mathbf{p}) \right)$$

It neglects spin, asymptotically small interactions only!



The method of moments converges fast

different treatment of the residual moments

$$\rho_{-2}^{\mu_1\cdots\mu_4} \to \rho_{-2}^{\mu_1\cdots\mu_4}\Big|_{eq.}$$

$$\rho_{-1}^{\mu_1\cdots\mu_4} \neq \rho_{-2}^{\mu_1\cdots\mu_4}\Big|_{eq.}$$

L T, G Vujnovich, J Noronha, U Heinz **arXiv:1808.06436**
L T, G Vujnovich **(WIP)**

Simplest case: free streaming

Off-shell non-trivial solutions!

$$W(x,k) = \delta(k^2 - m^2) W_{\text{on}}(x,k) = \delta(k^2 - m^2) \int \frac{d^4\xi}{(2\pi)^2} \, e^{ix\cdot\xi} \, \tilde{W}_{\text{on}}(\xi, k)$$

$$\hbar^2 \Box W(x,k) = 4\left(k^2 - m^2 c^2\right) W(x,k)$$

Incompatible constraints for the Fourier modes...

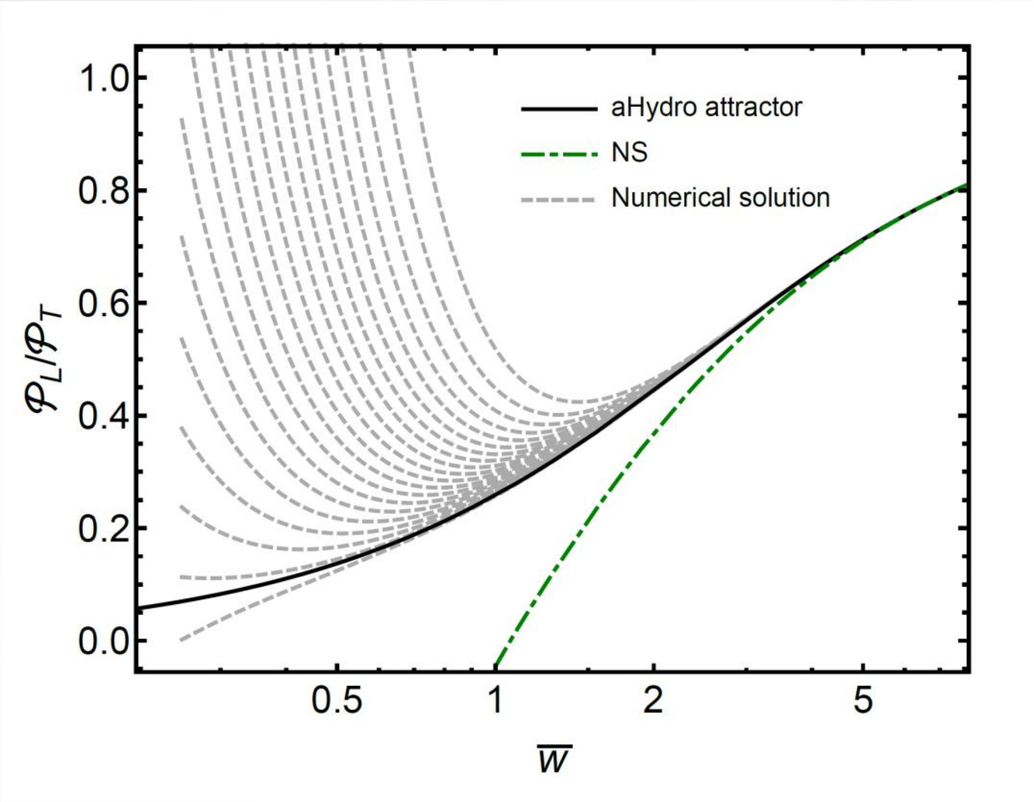
$$k \cdot \partial W(x,k) = 0$$

$$\xi^2 \, \tilde{W}_{\text{on}}(\xi, k) = 0$$

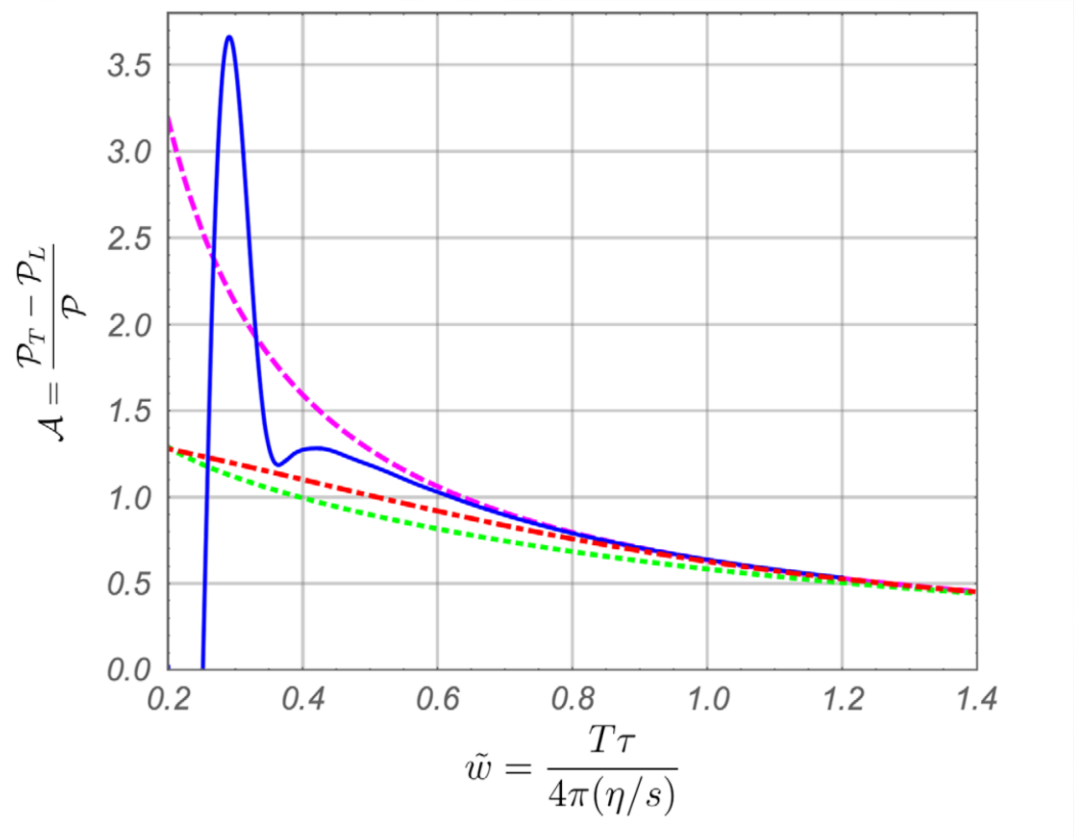
$$k \cdot \xi \, \tilde{W}_{\text{on}}(\xi, k) = 0$$

L T, Phys.Rev.D 108 (2023) 7, 076022

Attractor behavior already used to explain hydrodynamization



M Strickland, J Noronha, G Denicol
Phys. Rev. D 97, 036020 (2018)



M P. Heller, A Kurkela, M Spalinski, V Svensson
Phys. Rev. D 97, 091503 (2018)

Warning!
(symmetry of the expansion)

A Behtash, C N Cruz-Camacho, M Martinez, Phys. Rev. D 97, 044041 (2018)