Glueballs in the Hopfion approach

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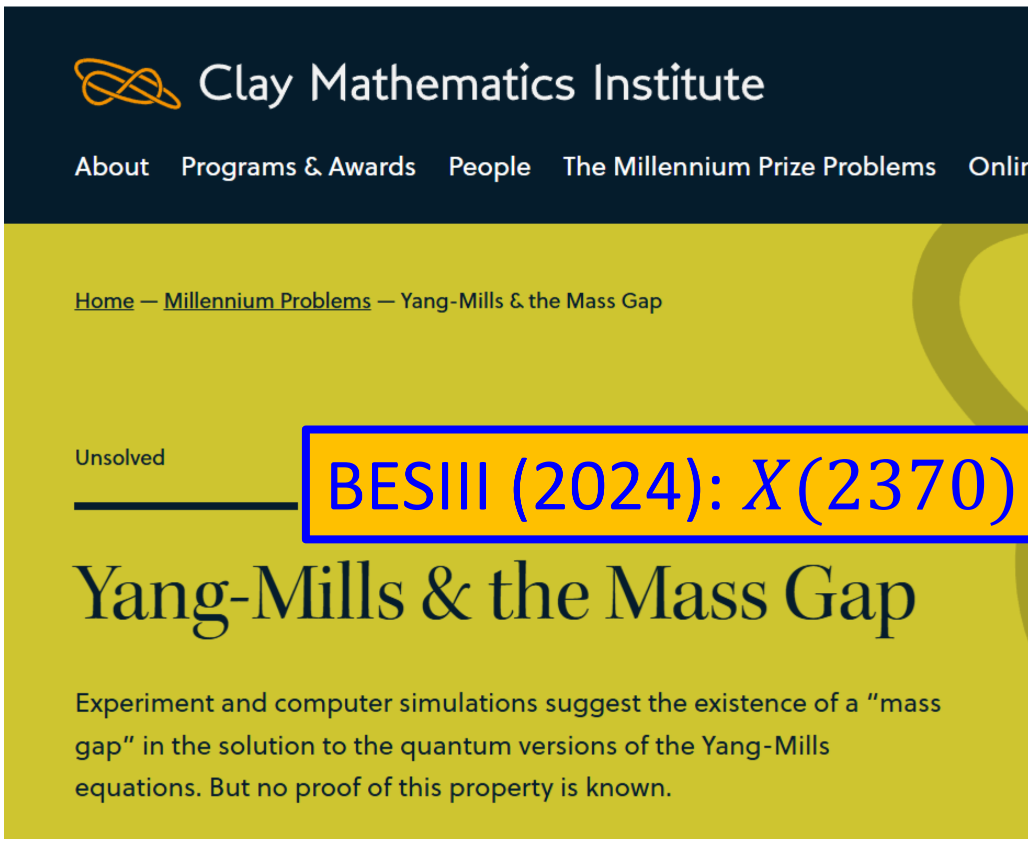
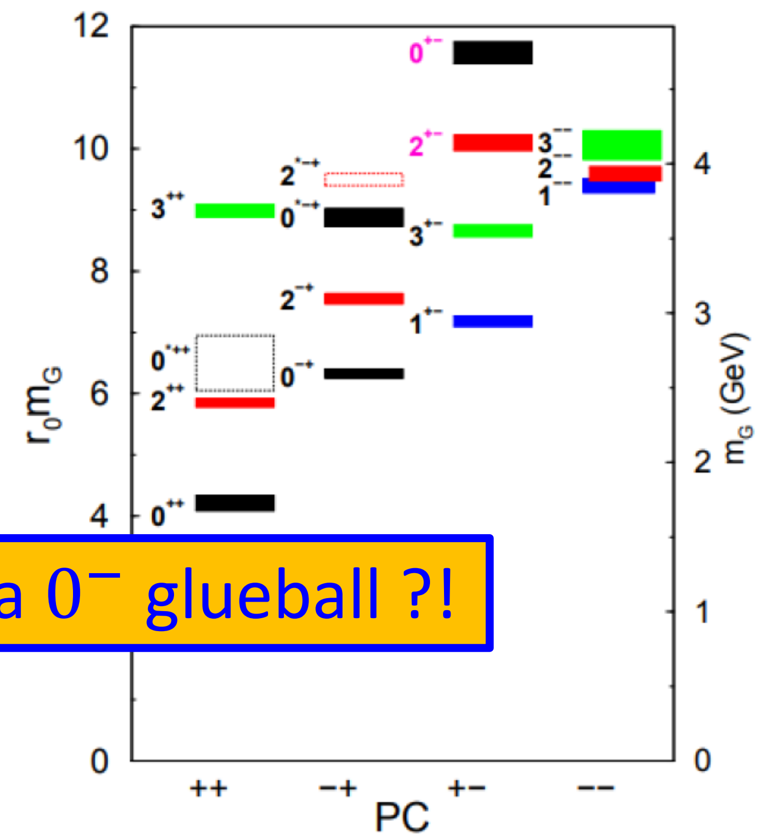
Why glueballs?

The key objects to understand the mass gap in pure Yang-Mills theory



BESIII (2024): $X(2370)$ as a 0^{-} glueball ?!



Morningstar & Peardon
PRD **60**, 034509 (1999)

The goal

Topological approach to non-perturbative physics

- ❑IR phenomenology of SU(2) Yang-Mills theory
- ❑Glueballs as topological solitons

    cf. Faddeev and Niemi (‘99), the lowest glueball as a Hopfion

➔ Higher-lying states and their structure, comparison to available experimental & lattice data

Based on the work in collaboration with Amari, Nitta, Sasaki, Shigaki, Yano and Yasui, Phys. Lett. B 869, 139805 (2025)

The Skyrme-Faddeev model (SFM)

A low-energy effective model of SU(2) YM

[Faddeev & Niemi (99)]

$$\mathcal{L} = \frac{\kappa^2}{4} \mathrm{Tr}\left(\partial_\mu \mathfrak{n} \partial^\mu \mathfrak{n}\right) + \frac{1}{32e^2} \mathrm{Tr}\left[\partial_\mu \mathfrak{n}, \partial_\nu \mathfrak{n}\right]^2$$

$$\mathfrak{n} = \boldsymbol{\tau} \cdot \boldsymbol{n} \qquad \boldsymbol{n} = (n_1, n_2, n_3) \text{ with } |\boldsymbol{n}| = 1$$

❑Topological solitons: Hopfions ← $\pi_3(S^2) = \mathbb{Z}$

❑Hopf charges: $Q = lm$ [$l, m \in \mathbb{Z}$ winding numbers]

❖Conjecture: glueballs as Hopfions

Energy eigenvalues

$$E = \frac{\kappa}{e} M^*_{\rm cl} + \frac{\kappa e^3}{2} \left\{ \frac{J(J+1)}{V^*_{11}} + \left(\frac{1}{U^*_{33}} - \frac{\ell^2}{V^*_{11}} \right) K_3^2 \right\}$$

Ansatz for a torus-shape Hopfion

$n(l, m; \theta, \varphi)$

$Q_{\rm top} = \ell m$	(ℓ, m)	$M^*_{\rm cl}$	V^*_{11}	U^*_{33}
1	(1,1)	274.79	372.17	227.78
2	(2,1)	447.20	968.33	267.72
2	(1,2)	508.16	1203.11	398.09

Phenomenological parameters

Energy specified by

- Winding numbers (l, m)
- Quantum numbers (J, K_3)
- Model parameters (κ, e)

Input (Q=1)

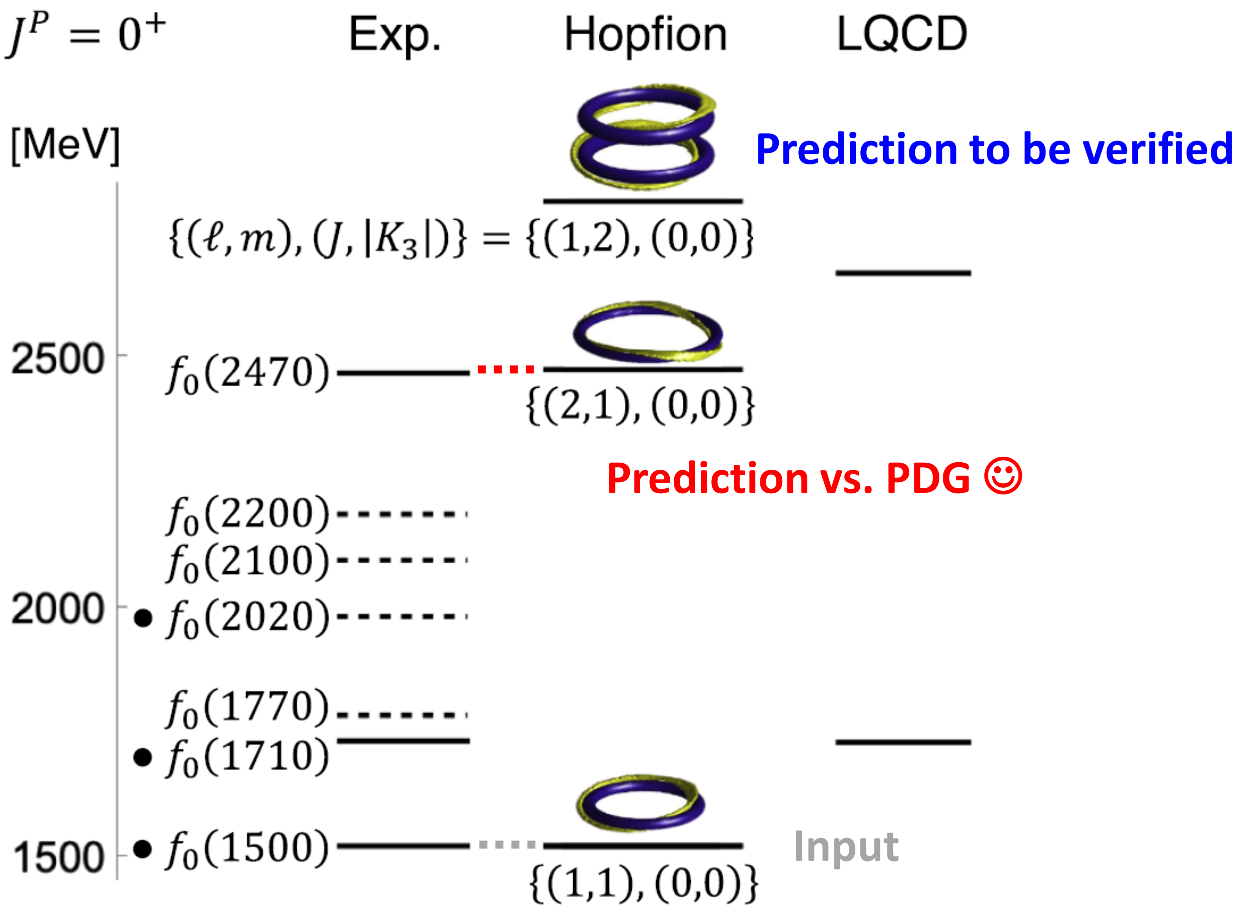
- J=0: $\{(l, m), (J, K_3)\} = \{(1,1), (0,0)\}$ as $f_0(1500)$

  cf. the lowest scalar glueball in Lattice QCD

- J=2: $\{(1,1), (2,0)\}$ as $f_J(2220)$ N.B. $\Gamma = 23$ MeV!

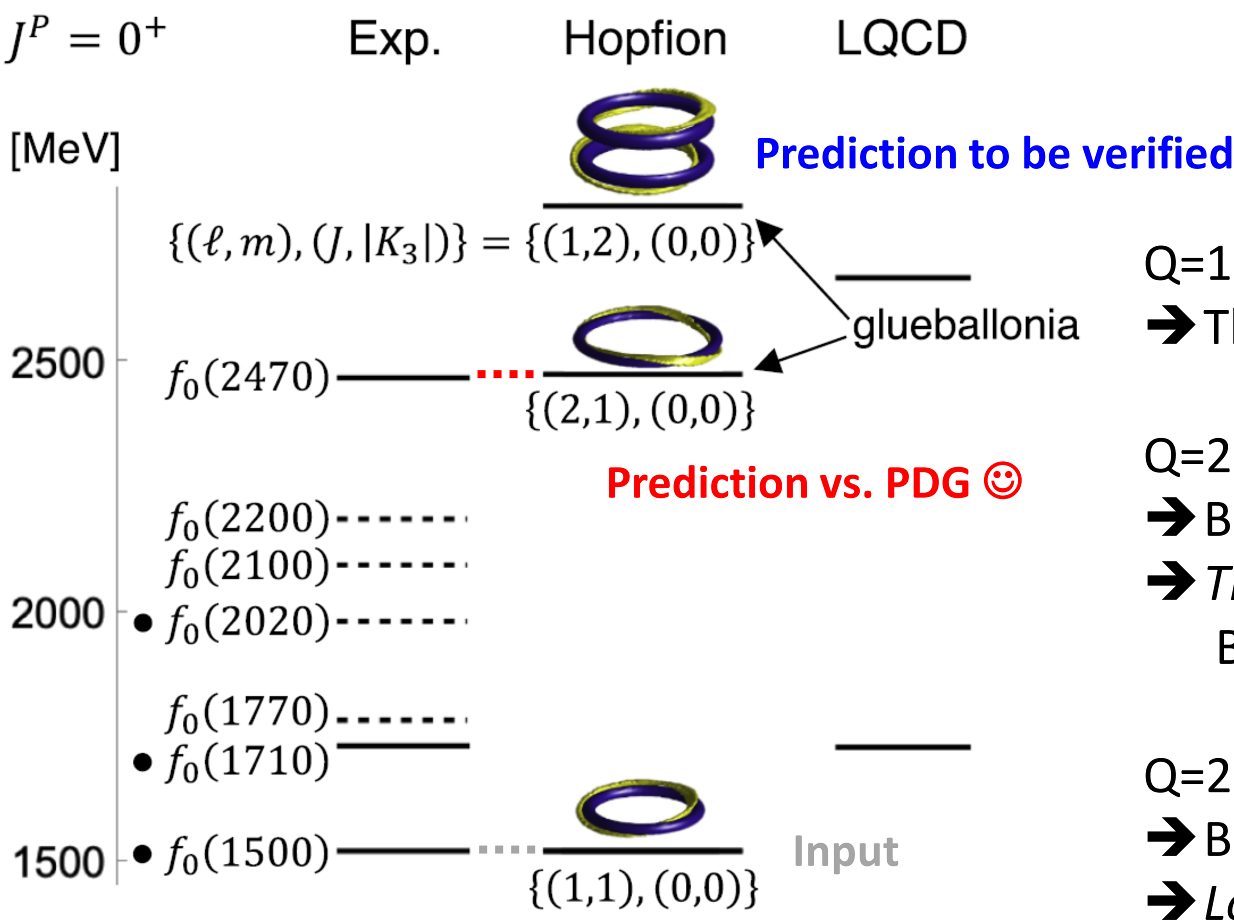
$$\kappa/e = 5.54\ \mathrm{MeV}, \quad \kappa e^3 = 88.5\ \mathrm{GeV}$$

Mass spectrum of scalar glueballs



● Listed as established states in PDG
Solid: narrow states ($\Gamma < 150$ MeV)
Broken: broad states ($\Gamma > 150$ MeV)

Mass spectrum of scalar glueballs



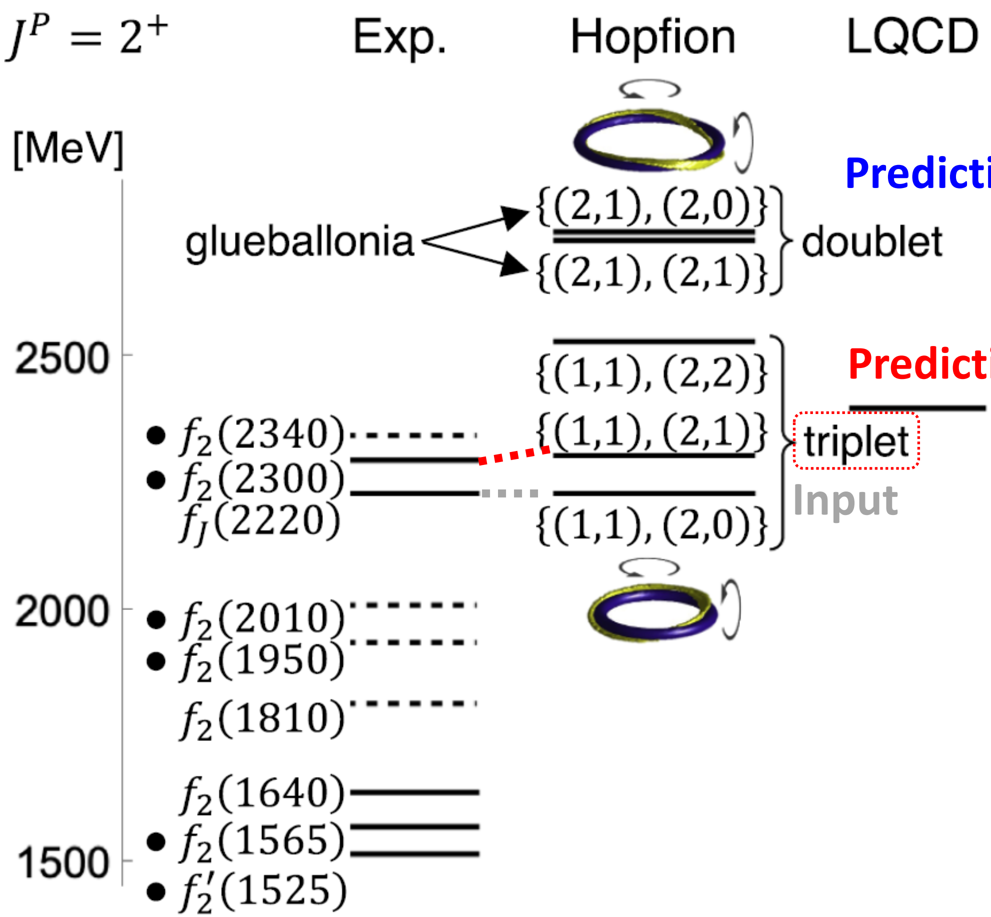
Q=1 Hopfion
➔ The lowest glueball

Q=2 Hopfion w/ $(l, m) = (2,1)$
➔ Bound state of 2 Q=1 Hopfions
➔ *Tightly* bound glueballonium
B.E. = 577 MeV

Q=2 Hopfion w/ $(l, m) = (1,2)$
➔ Bound state of 2 Q=1 Hopfions
➔ *Loosely* bound glueballonium
B.E. = 230 MeV

● Listed as established states in PDG
Solid: narrow states ($\Gamma < 150$ MeV)
Broken: broad states ($\Gamma > 150$ MeV)

Mass spectrum of tensor glueballs



Prediction to be verified

Prediction vs. PDG ☺

Q	(ℓ, m)	multiplet	$(J, \|K_3\|)$	E [MeV]	Exp. [MeV]
1	(1,1)	singlet	(0,0)	1522	1500*
		triplet	(2,0)	2231	2220*
			(2,1)	2306	2300
			(2,2)	2531	2340
2	(2,1)	singlet	(0,0)	2477	2470
		doublet	(2,0)	2749	—
			(2,1)	2732	—
2	(1,2)	singlet	(0,0)	2814	—
		triplet	(2,0)	3034	—
			(2,1)	3108	—
			(2,2)	3330	—

• Listed as established states in PDG
Solid: narrow states ($\Gamma < 150$ MeV)
Broken: broad states ($\Gamma > 150$ MeV)

Remarks

- ❑ Non-topological approach
  - A model for dilatons [Giacosa, Pilloni & Trotti, ('22)]
  - The lowest glueballonium of the mass 3.4 GeV
  - cf. our Hopfion approach predicts 2.4 GeV

On our agenda:

- ❑ Extension to color SU(3)
- ❑ Coupling to light quarks

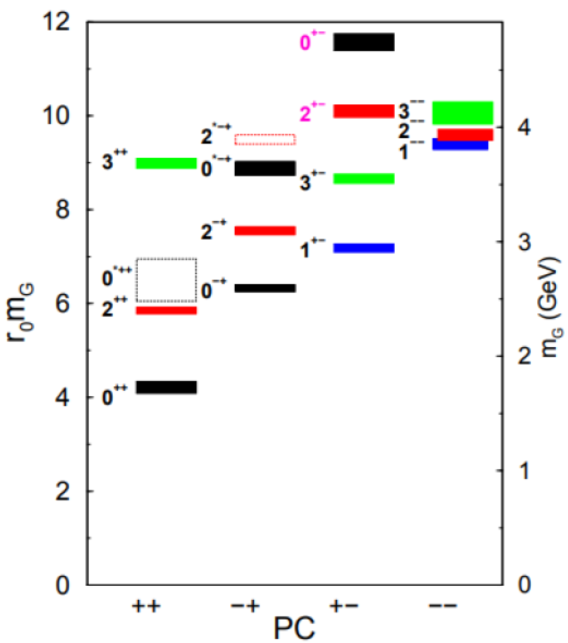
Summary

- ❑ $f_0(2470)$: high mass & narrow ($\Gamma = 75$ MeV)
  ➔ glueballonium, B.E.=577 MeV
- ❑ $f_J(2200)$: another mysterious massive narrow state ($\Gamma = 23$ MeV)
  ➔our conjecture: spin-2 stable glueball
  - Characteristic multiplets of glueballonia
  - Their internal structures and binding energies
- ❖Pseudo-scalar glueballs as a benchmark test

  [Amari et al., in progress]

  - So far, only rotational modes
  - Vibration modes of a Hopfion → excited states
  - vs. BESIII $X(2370)$ as a pseudo-scalar glueball

Backup



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0^{++} light unflavored mesons from PDG

	Mass[MeV]	Width[MeV]
● $f_0(980)$	990	$10-100$
● $f_0(1370)$	$1200-1500$	$200-500$
● $f_0(1500)$	1522	108
● $f_0(1710)$	1733	150
$f_0(1770)$	1784	161
● $f_0(2020)$	1982	440
$f_0(2100)$	2095	287
$f_0(2200)$	2187	210
$f_0(2330)$		
$f_0(2470)$	2470	75

● indicates established particles

2^{++} light unflavored mesons from PDG

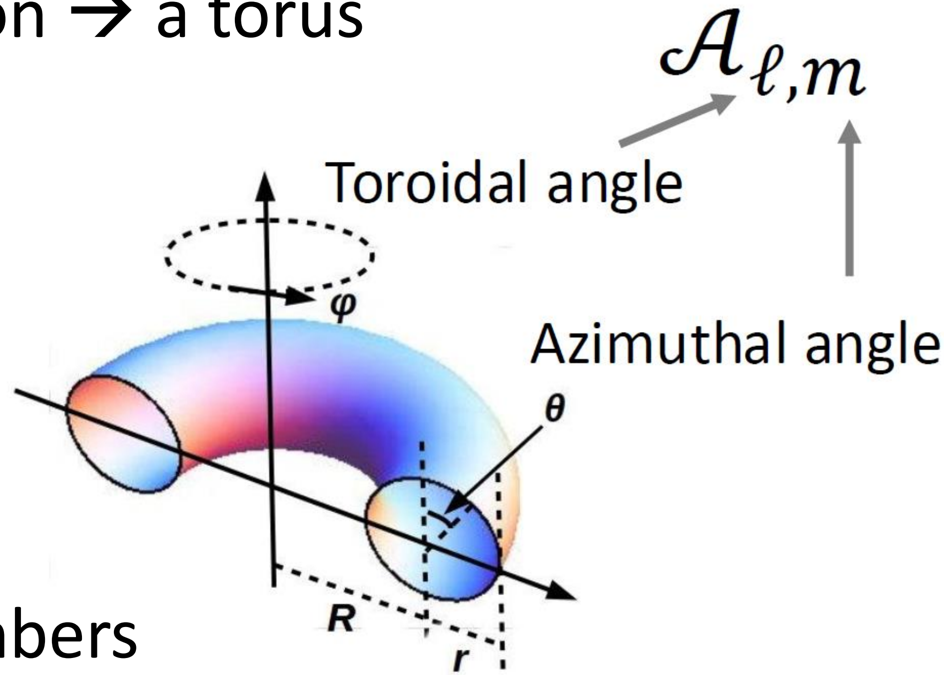
	Mass[MeV]	Width[MeV]
● $f_2(1270)$	1275	186
$f_2(1430)$	1430	
● $f_2'(1525)$	1517	72
● $f_2(1565)$	1571	132
$f_2(1640)$	1639	100
$f_2(1810)$	1815	197
$f_2(1910)$		
● $f_2(1950)$	1936	464
● $f_2(2010)$	2010	200
$f_2(2150)$		
$f_J(2220)$	2231	23
● $f_2(2300)$	2297	150
● $f_2(2340)$	2346	331

The Skyrme-Faddeev model (SFM)

An axial-symmetric Hopfion → a torus

[Gladikowski & Hellmund (97)]

[Battye & Sutcliffe (98)]



- ❑Hopf charges: $Q = lm$

  $l, m \in \mathbb{Z}$ winding numbers

- ❑Static energy configurations via rational map

Energy eigenvalues

- Classical mass [$M_{\mathrm{cl}} = \frac{\kappa}{e} M_{\mathrm{cl}}^*$]
- Inertia tensors [$X_{jk} = \frac{1}{\kappa e^3} X_{jk}^*$]

$$M_{\mathrm{cl}}^* = \int d^3x \left[\frac{1}{2}(\partial_i \boldsymbol{n})^2 + \frac{1}{4}\left(\partial_i \boldsymbol{n} \times \partial_j \boldsymbol{n}\right)^2\right]$$

$$U_{jk}^* = \int d^3x \left[\delta_{jk} - n_j n_k + \partial_l n_j \partial_l n_k\right]$$

$$V_{jk}^* = \int d^3x \left[\left(i\mathrm{L}_j \boldsymbol{n}\right) \cdot (i\mathrm{L}_k \boldsymbol{n}) + \left(i\mathrm{L}_j \boldsymbol{n} \times \boldsymbol{n}\right) \cdot (i\mathrm{L}_k \boldsymbol{n} \times \boldsymbol{n})\right]$$

with $\mathrm{L}_j = -i\varepsilon_{jkl} x^k \partial_l$