From novel Wigner function for spin-1/2 to causal and stable spin hydrodynamics

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Based on arXiv:2505.02657, arXiv:2511.09580, arXiv:2511.19295

In collaboration with Samapan Bhadury, Zbigniew Drogosz,

Wojciech Florkowski and Sudip Kumar Kar

XVIII Polish Workshop on Relativistic Heavy-Ion Collisions

Describing particles with spin 1/2

Spin density matrices $f^{\pm}(x,p)$ (2×2) & spinor density matrices $X^{\pm}(x,p)$ (4×4):

$$f^{+}_{rs}(x,p) = \frac{1}{2m}\,\bar{u}_r(p) X^{+} u_s(p), \qquad f^{-}_{rs}(x,p) = -\frac{1}{2m}\,\bar{v}_s(p) X^{-} v_r(p), \qquad r,s = \{1,2\}$$

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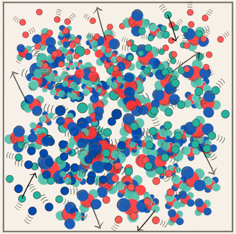

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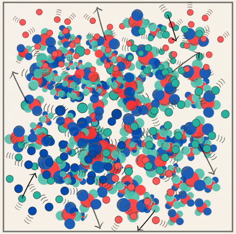

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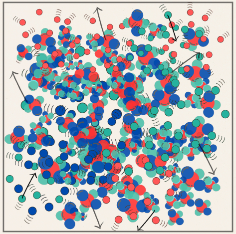

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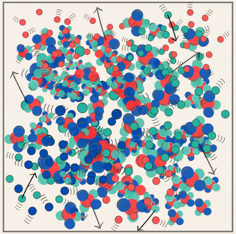

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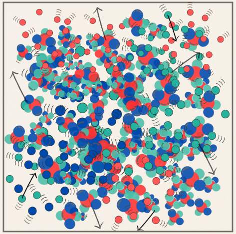

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$$W^{\pm}(x,k) = \frac{1}{4m}\int \mathrm{d}P\,\delta^{(4)}(k\mp p)(\not{p}\pm m)X^{\pm}(\not{p}\pm m) \tag{1}$$

[De Groot, Relativistic Kinetic Theory. Principles and Applications (1980)]

Spinor density matrix $X^{\pm}$

Commonly used form of $X^{\pm}$ in local equilibrium

[Becattini et al., Annals Phys. 338 (2013)]

$$X^{\pm} = \exp\left[-\frac{p^{\mu}u_{\mu}}{T} \pm \frac{\mu}{T}\right] \exp\left[\pm\frac{1}{2}\omega_{\mu\nu}(x)\Sigma^{\mu\nu}\right] \tag{2}$$

$$\Sigma^{\mu\nu} = \tfrac{i}{4}[\gamma^{\mu}, \gamma^{\nu}]$$

$\omega_{\mu\nu} = \Omega_{\mu\nu}/T$ – spin polarization tensor, $\Omega_{\mu\nu}$ – spin chemical potential

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$$\omega_{\mu\nu} = \begin{bmatrix} 0 & e^1 & e^2 & e^3 \\ -e^1 & 0 & -b^3 & b^2 \\ -e^2 & b^3 & 0 & -b^1 \\ -e^3 & -b^2 & b^1 & 0 \end{bmatrix} \tag{3}$$

$\boldsymbol{e} = (e^1, e^2, e^3)$, $\boldsymbol{b} = (b^1, b^2, b^3)$ – electric- and magnetic-like vectors

Motivation

Mean spin polarization of spin-1/2 particles in equilibrium, $\mu = 0$

[Florkowski et al. PRD 97, 116017 (2018)]

$$\boldsymbol{P} = \frac{1}{2}\frac{\mathrm{tr}_2\left(f^{\pm}\boldsymbol{\sigma}\right)}{\mathrm{tr}_2\left(f^{\pm}\right)} = -\frac{1}{2}\tanh\left[\frac{\sqrt{\boldsymbol{b}_* \cdot \boldsymbol{b}_* - \boldsymbol{e}_* \cdot \boldsymbol{e}_*}}{2}\right]\frac{\boldsymbol{b}_*}{\sqrt{\boldsymbol{b}_* \cdot \boldsymbol{b}_* - \boldsymbol{e}_* \cdot \boldsymbol{e}_*}} \tag{4}$$

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+ *other issues*

Deriving $X^{\pm}$ from the spin density matrices

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Decomposition of 2×2 Hermitian matrices $f^{\pm}_{rs}(x,p)$ [Florkowski et al., PRD97 (2018)]:

$$f^{\pm}_{rs}(x,p) = f^{\pm}_{0}(x,p)\left[\delta_{rs} + \boldsymbol{\zeta}^{\pm}_{*}(x,\boldsymbol{p}) \cdot \boldsymbol{\sigma}_{rs}\right], \qquad r,s = \{1,2\} \tag{7}$$

- $f^{\pm}_{0}(x,p)$ – spin-averaged phase-space density
- $\zeta^{\pm\mu}_{*} = \left(0, \boldsymbol{\zeta}^{\pm}_{*}\right)$ – mean polarization vector in PRF, $\quad 0 \leq |\boldsymbol{\zeta}^{\pm}_{*}| \leq 1$

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$$\bar{u}_r(p)\gamma_5 \zeta^{+}_{\mu}\gamma^{\mu} u_s(p) = 2m\,\boldsymbol{\zeta}^{+}_{*}\cdot\boldsymbol{\sigma}_{rs} \tag{8}$$

$$\bar{v}_s(p)\gamma_5 \zeta^{-}_{\mu}\gamma^{\mu} v_r(p) = -2m\,\boldsymbol{\zeta}^{-}_{*}\cdot\boldsymbol{\sigma}_{rs} \tag{9}$$

New spinor density $X_s^{\pm}$

$$\Longrightarrow \; X_s^{\pm}(x,p) = f_0^{\pm}(x,p)\left[1 + \gamma_5 \not{\zeta}^{\pm}(x,p)\right] \tag{10}$$

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For general spacelike four-vectors a^μ satisfying $a^2 < 0$

$$\exp(\gamma_5 \not{a}) = \cosh\sqrt{-a^2}\left[1 + \frac{\gamma_5 \not{a}}{\sqrt{-a^2}} \tanh\sqrt{-a^2}\right] \tag{11}$$

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$$\implies X_s^\pm(x,p) = f_0^\pm(x,p)\left[1+\gamma_5 \not{\zeta}^\pm(x,p)\right] \tag{10}$$

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Equilibrium spinor density for Boltzmann statistics

$$\boxed{X_s^\pm(x,p) = \exp\left(-\frac{p_\mu u^\mu}{T} \pm \frac{\mu}{T}\right)\exp\left(\gamma_5 \not{a}\right)}, \qquad \zeta_\pm^\mu = a^\mu \tag{12}$$

$$a_\mu(x,p) = -\frac{1}{2m}\tilde{\omega}_{\mu\nu}(x)p^\nu, \qquad \tilde{\omega}_{\mu\nu} = \frac{1}{2}\,\epsilon_{\mu\nu\alpha\beta}\,\omega^{\alpha\beta} \tag{13}$$

Comparison of spinor densities

Previously known expression [Becattini, Chandra, Del Zanna, Grossi, Annals Phys. 338 (2013)]

$$X^{\pm}(x,p) = \exp\left[-\frac{p^{\mu}u_{\mu}}{T} \pm \frac{\mu}{T}\right] \exp\left[\pm\frac{i}{8}\omega_{\mu\nu}(x)[\gamma^{\mu},\gamma^{\nu}]\right] \tag{14}$$

Our new finding [Bhadury, Drogosz, Florkowski, Kar, VM, arXiv:2505.02657]

$$X_s^{\pm}(x,p) = \exp\left[-\frac{p^{\mu}u_{\mu}}{T} \pm \frac{\mu}{T}\right] \exp\left[-\frac{1}{4m}\gamma_5\,\epsilon_{\mu\nu\alpha\beta}\,\omega^{\alpha\beta}(x)\,p^{\nu}\,\gamma^{\mu}\right] \tag{15}$$

New spinor densities – various distributions

From Boltzmann

$$X^{\pm}_{\alpha\beta} = \frac{1}{2}\left[\left(g_{\mathrm{B}}^{\pm +} + g_{\mathrm{B}}^{\pm -}\right)\delta_{\alpha\beta} + \frac{(\gamma_5 \not{a})_{\alpha\beta}}{\sqrt{-a^2}}\left(g_{\mathrm{B}}^{\pm +} - g_{\mathrm{B}}^{\pm -}\right)\right] \tag{16}$$

$$g_{\mathrm{B}}^{\pm\pm}(x,p) = \exp\left[-\frac{p^{\mu}u_{\mu}}{T} \pm \frac{\mu}{T} \pm \sqrt{-a^2(x,p)}\right] \tag{17}$$

to Fermi–Dirac statistics

$$X^{\pm}_{\alpha\beta} = \frac{1}{2}\left[\left(g_{\mathrm{FD}}^{\pm +} + g_{\mathrm{FD}}^{\pm -}\right)\delta_{\alpha\beta} + \frac{(\gamma_5 \not{a})_{\alpha\beta}}{\sqrt{-a^2}}\left(g_{\mathrm{FD}}^{\pm +} - g_{\mathrm{FD}}^{\pm -}\right)\right] \tag{18}$$

$$g_{\mathrm{FD}}^{\pm\pm}(x,p) = \left[\exp\left(\frac{p^{\mu}u_{\mu}}{T} \mp \frac{\mu}{T} \mp \sqrt{-a^2(x,p)}\right) + 1\right]^{-1} \tag{19}$$

Spinor density application

$$W^{\pm}(x,k) = \frac{1}{4m}\int \mathrm{d}P\,\delta^{(4)}(k \mp p)(\not{p} \pm m)\,X^{\pm}(\not{p} \pm m) \to \text{thermodynamics!}$$

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$$N^{\mu}(x) = \sum_{r=1}^{2} \int \mathrm{d}P\, p^{\mu} \left[f^{+}_{rr}(x,p) - f^{-}_{rr}(x,p)\right] - \textit{baryon current} \tag{20}$$

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$$T^{\mu\nu}(x) = \sum_{r=1}^{2}\int \mathrm{d}P\,p^\mu p^\nu\left[f^{+}_{rr}(x,p) + f^{-}_{rr}(x,p)\right] - \textit{energy-momentum tensor} \tag{21}$$

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$$S^{\lambda,\mu\nu}(x) = \frac{1}{2}\sum_{r,s=1}^{2}\int \mathrm{d}P\,p^{\lambda}\left[\sigma^{+\mu\nu}_{sr}(p)f^{+}_{rs}(x,p) + \sigma^{-\mu\nu}_{sr}(p)f^{-}_{rs}(x,p)\right] - \textit{spin tensor}$$

Coincide with classical-spin kinetic expressions [Florkowski et al. PRD 97, 116017 (2018)]

Thermodynamic currents

$T^{\mu\nu}$, N^{μ}, S^{μ}, $S^{\lambda,\mu\nu}$... – from $X^{\pm}$ or from the **generating function**
(next talk by Zbigniew Drogosz)

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Generating function indicates that our *perfect hydrodynamic* theory is of the **divergence-type**

+ *non-linearly causal* and **stable**

Profits: built-in conservation laws, physically meaningful, numerically consistent predictions from hydro simulations.

[Abboud, Gavassino, Singh, Speranza, PRD 112 (2025); Bhadury, Drogosz, Florkowski, Kar, VM, 2511.19295]

Summary

We have constructed:

- Wigner function for spin-1/2 particles in local equilibrium,
- accounting for a proper normalization of the mean polarization vector,
- incorporating either Boltzmann or Fermi–Dirac statistics,
- leading to a nonlinearly causal and stable theory.

Perspectives: spin-orbit interaction included as dissipation, spin-1 particles, HIC data description.

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