Magnetic properties of hot hadronic medium

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in collaboration with Wojciech Broniowski

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Institute of Nuclear Physics, PAN, Krakow

XVIII Polish Workshop on Relativistic Heavy-Ion Collisions

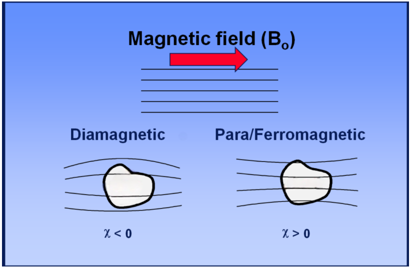
NATIONAL SCIENCE CENTRE POLAND

Magnetic Susceptibility

- **Magnetic response of a medium to the applied magnetic field,**

$$\chi_B = \frac{\partial \mathcal{M}}{\partial B}\bigg|_{B=0}$$

$\mathcal{M} \rightarrow$ Magnetization of the system



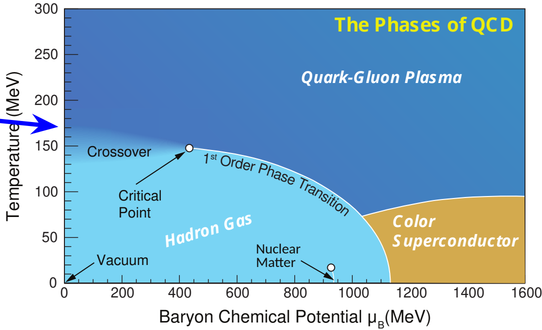
Thermodynamic system under magnetic field

- The grand potential: $\Omega = U - TS - B\mathcal{M} - \mu_{\rm ch} N = -P\,V$.
- Magnetization and magnetic susceptibility :

$$\mathcal{M} = \frac{\partial P}{\partial B} \quad \text{and} \quad \chi_B = \frac{\partial^2 P}{\partial B^2}\bigg|_{B=0}$$

Our goal

- At LHC and RHIC highest energy, $\mu_B \approx 0 \longrightarrow$ smooth crossover transition, $T_c \sim 155$ MeV



Stephanov *et al.* (1998), PRL; Aoki *et al.* (2006), Nature

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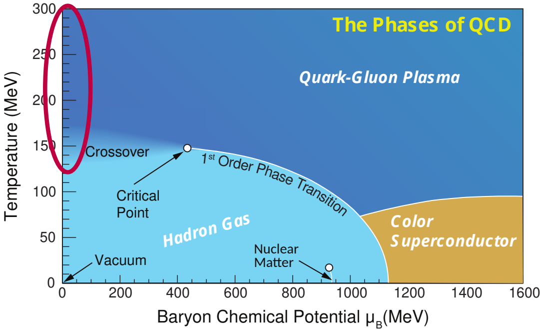

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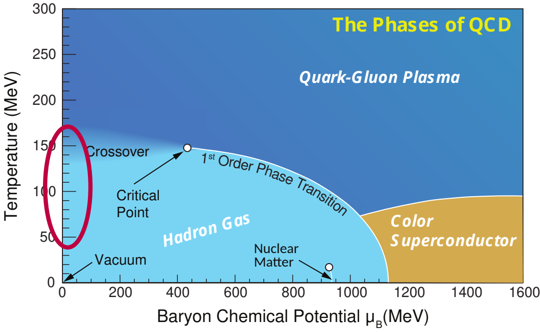

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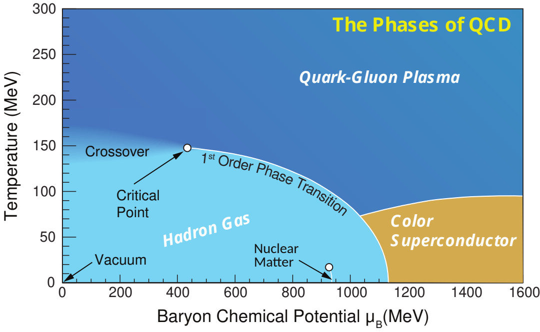

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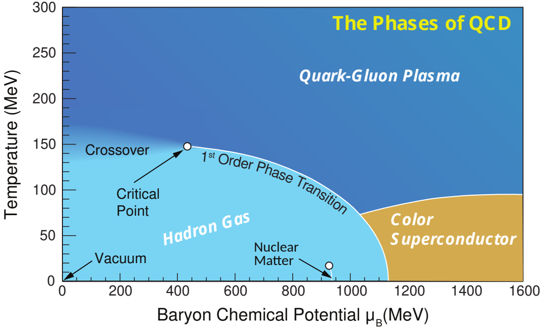

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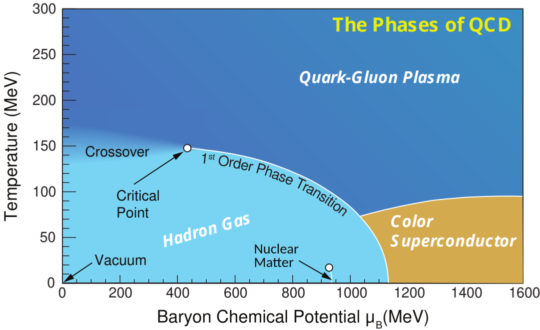

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- Importance of vacuum contribution and anomalous magnetic moment.

Charged particle in a magnetic field

- Relativistic energy :

$$E = \left(M^2 + p_z^2 + \underbrace{B|Q|(2l+1)}_{\text{Landau diamagnetism}} \underbrace{-2QBs_z}_{\text{Pauli paramagnetism}} \right)^{1/2}$$

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$$P^{\rm th} = -\eta T \frac{B|Q|}{2\pi^2} \sum_{l=0}^{\infty} \sum_{s_z} \int_0^{\infty} dp_z \log[1 - \eta\ f]$$

where $f(E(B,p_z),\mu_{ch},T) = \frac{1}{\exp\left(\frac{E(B,p_z)-\mu_{ch}}{T}\right)+\eta}$, $\quad \mu_{ch} = \mu_{\mathcal{B}}\mathcal{B} + \mu_{\mathcal{S}}\mathcal{S} + \mu_{\mathcal{Q}}\mathcal{Q}$

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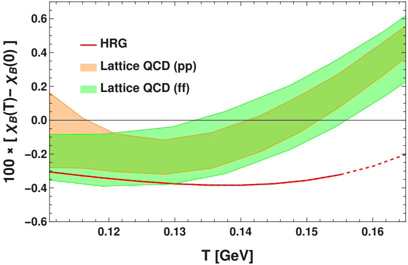
- Magnetic susceptibility:

$$\chi_B^{\rm th} = \frac{\partial^2 P^{\rm th}}{\partial B^2}|_{B=0} = \begin{cases} + & \text{if } s \neq 0 : \text{ Paramagnetic} \quad \text{e.g } p, \rho, \Delta \\ - & \text{if } s = 0 : \text{ Diamagnetic} \quad \text{e.g. } \pi, K \end{cases}$$

Magnetic Susceptibility (χ_B): HRG vs lattice QCD

- The magnetic susceptibility in HRG:

$$\chi_B^{HRG} = \sum_{\text{states}} \chi_B^{\text{th}}$$



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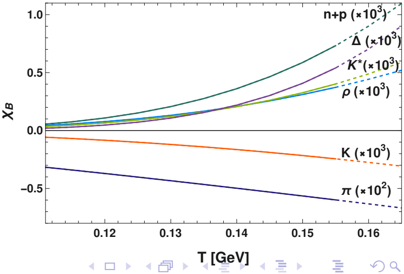

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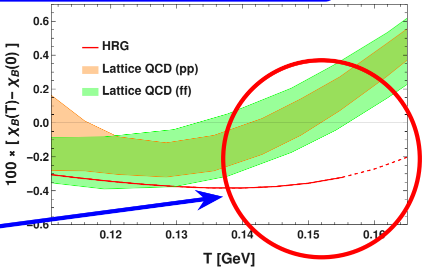

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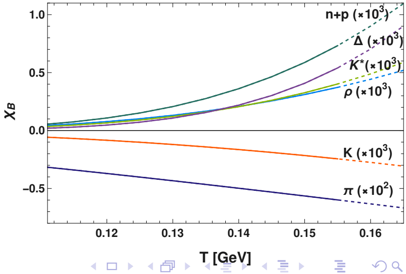

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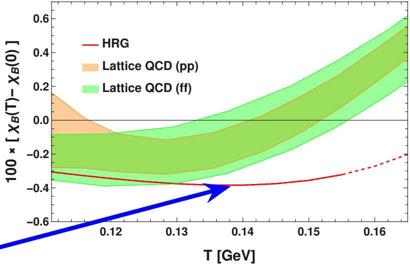

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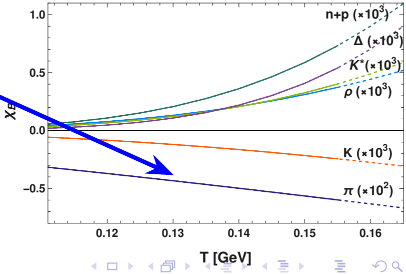

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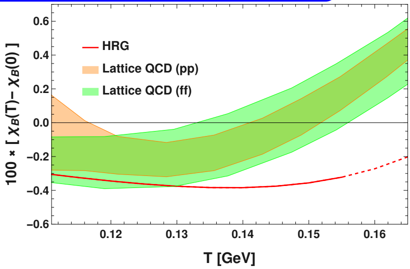

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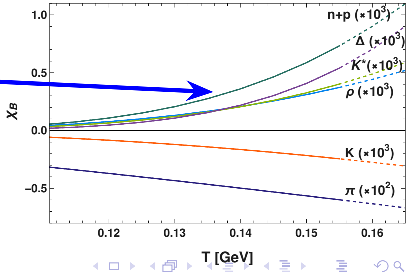

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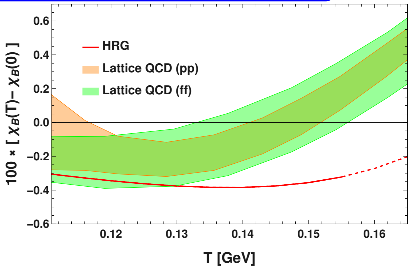
- **Genuine need for light paramagnetic source (e.g. quarks?)**



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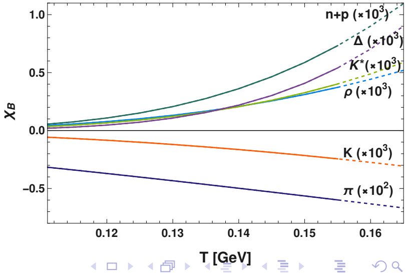
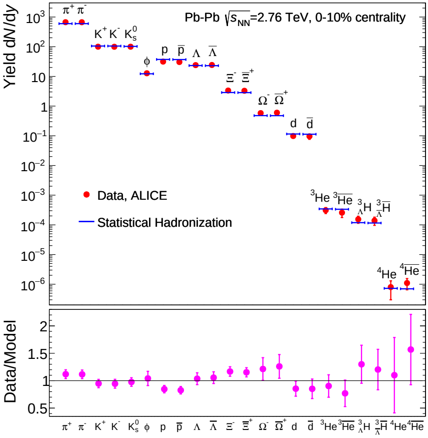


Successes of HRG

- Remarkably successful to reproduce particle multiplicity seen in heavy-ion collision



Andronic *et al.* (2018), Nature

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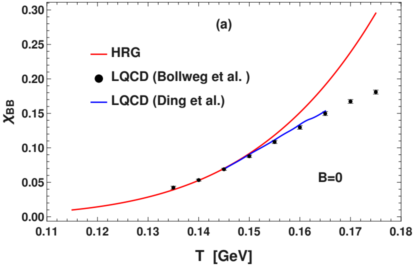
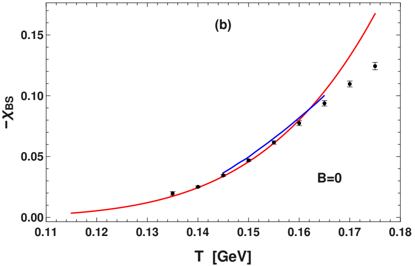


Bollweg *et al.* (HotQCD) (2021), PRD



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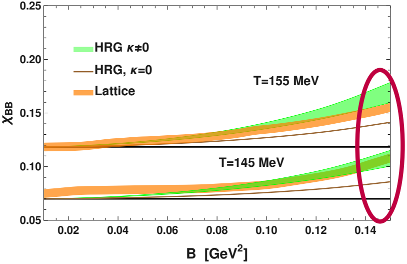
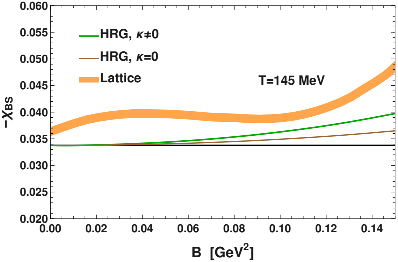


RS *et al.* (2025), PRC; Ding *et al.* (2024), PRL



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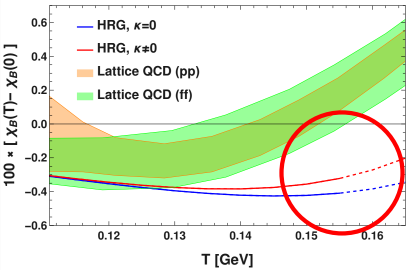
- Non-zero κ of hadrons are not sufficient for χ_B



We address this failure as a genuine problem

Importance of vacuum pressure

- The total pressure can be decomposed:

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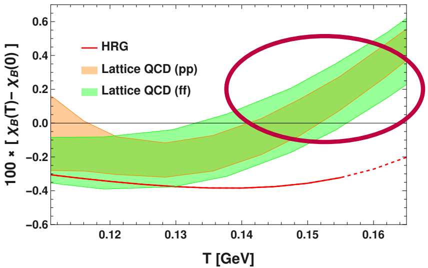
- Explicit B-dependence of P^{vac}:

$$\chi_B^{\text{vac}} \neq 0$$

Quark degrees of freedom: Quark-Meson model

Model

- We need paramagnetism (but also light): constituent quarks



Temp-dependent quark masses

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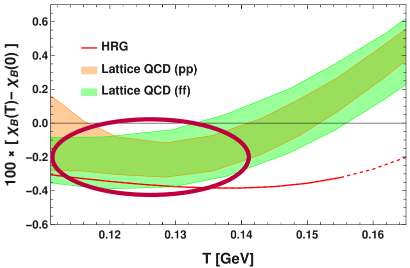

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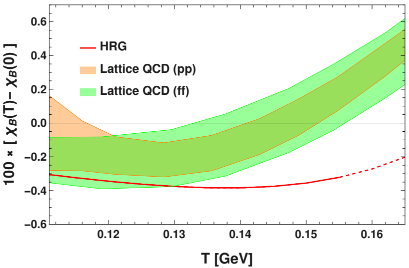

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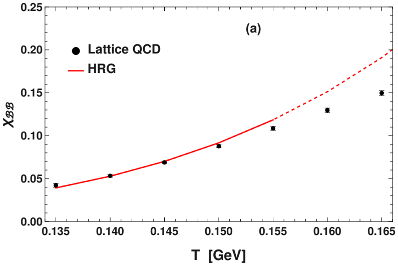

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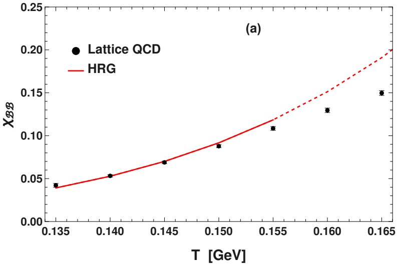

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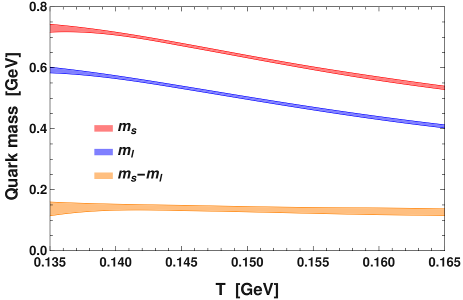

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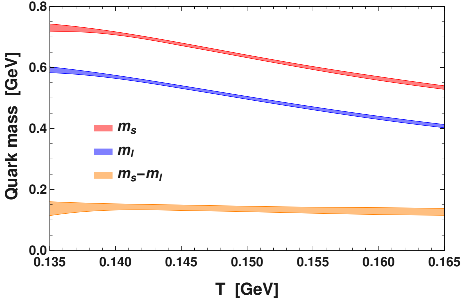

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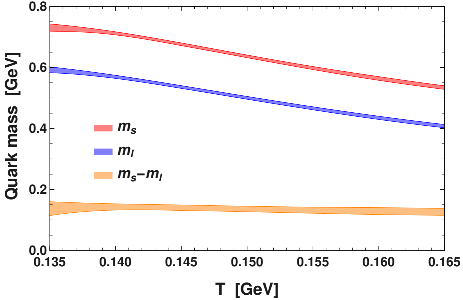

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- $m_s - m_l$ is roughly constant ~ 120 MeV

The vacuum contributions of quarks and mesons

Endrodi ***et al.*** **(2013), JHEP; Kamikado** ***et al.*** **(2015), JHEP**

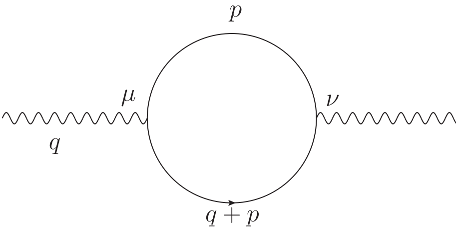
- Can be calculated from photon vacuum polarization involving a quark/pion loop ($\Pi^{\mu\nu}$).

$$\chi_B^{\text{vac}} = \frac{1}{2} \lim_{\vec{q}\to 0} \lim_{q_0 \to 0} \frac{\partial^2 \text{Re}[\Pi_s^{\text{vac}}]}{\partial q^2}$$

where $\Pi_s^{\text{vac}} = \frac{1}{2} \sum_{i=1}^{3} \Pi_{ii}^{\text{vac}}$

Coupling with anomalous magnetic moment (κ)

$$\Gamma^\mu = \gamma^\mu + \frac{\kappa}{2m} \sigma^{\mu\nu} q^\nu$$



Photon vacuum polarization (quark loop)

Endrodi *et al.* (2022), JHEP

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where $\Pi_s^{\text{vac}} = \frac{1}{2} \sum_{i=1}^{3} \Pi_{ii}^{\text{vac}}$

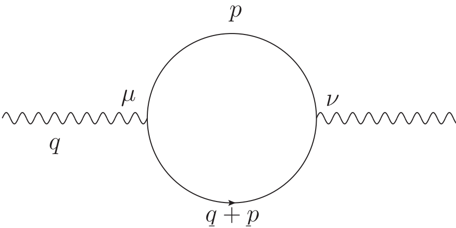
- After regularizing the divergence:

$$\chi_B^{\text{vac}}(T) - \chi_B^{\text{vac}}(0) = f(m(T), m(0), \kappa, \Lambda)$$

$\Lambda \sim 800$ MeV $\longrightarrow$ regularization scale

Coupling with anomalous magnetic moment (κ)

$$\Gamma^\mu = \gamma^\mu + \frac{\kappa}{2m} \sigma^{\mu\nu} q^\nu$$

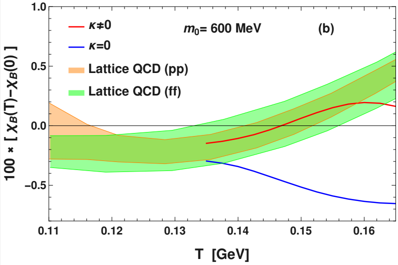


Photon vacuum polarization (quark loop)

Endrodi *et al.* (2022), JHEP

Result for χ_B in Quark-Meson model

- Temperature dependent quark and meson masses: $m(T)$, Kamikado *et al.* (2015), JHEP



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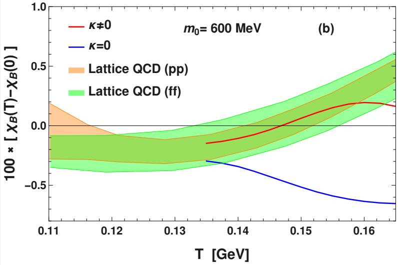

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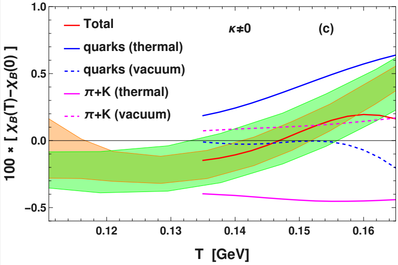

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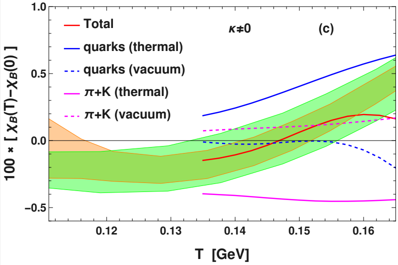

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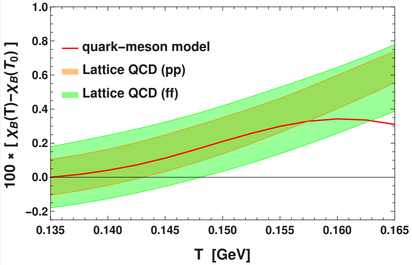
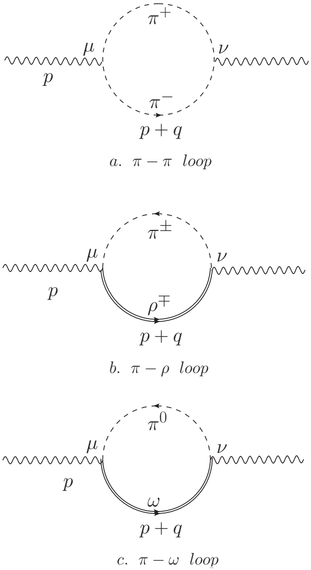
- Accurate knowledge of $m_l(0)$ is missing $\longrightarrow$ **the model reproduces χ_B subtracted at $T_0 = 135$ MeV**



Contribution of π-vector meson loop

- In hadronic picture: pion-vector meson loop in $\Pi^{\mu\nu}$.



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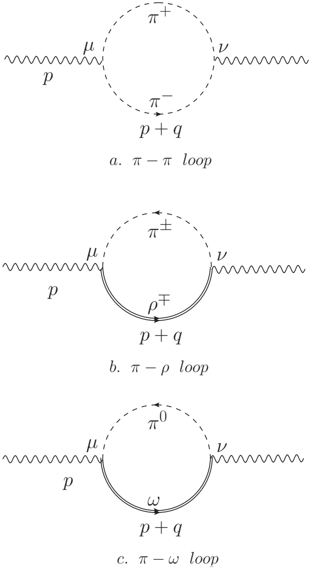

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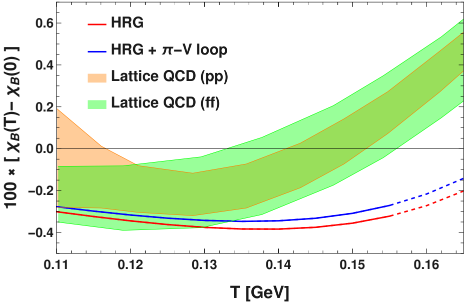

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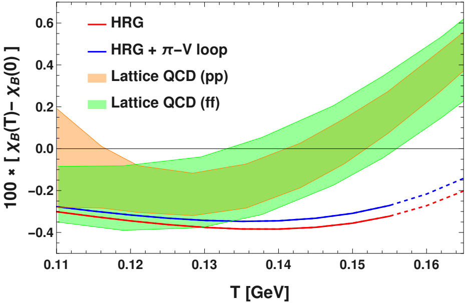
- **The effect is small and paramagnetic**
- In comparison to HRG , the contribution is at the level of 10-20 % $\longrightarrow$ comparable to two-loop χPT calculation.



Conclusions

- HRG cannot describe simultaneously the lattice data for $\chi_{\mathcal{BB}}$, $\chi_{\mathcal{BS}}$ and χ_B (a low-B feature) $\longrightarrow$ genuine need for light paramagnetic state below T_c

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Conclusions

- HRG cannot describe simultaneously the lattice data for $\chi_{\mathcal{BB}}$, $\chi_{\mathcal{BS}}$ and χ_B (a low-B feature) $\longrightarrow$ genuine need for light paramagnetic state below T_c
- Non-interacting quark-meson framework $\longrightarrow$ larger constituent quark-mass (quasi-particle picture)
- **The vacuum contribution directly affects χ_B** $\longrightarrow$ need better modeling
- **The quark-meson model can successfully describe lattice data for χ_B while simultaneously reproducing $\chi_{\mathcal{BB}}$ and $\chi_{\mathcal{BS}}$**

Thank you !

Backup

Self-consistency requirement

- Charge-conjugation symmetry $\longrightarrow$ particle and anti-particle have same energy:

$$P^{\rm th}(T,\mu,B;\{m\}) = P^{\rm th}(T,-\mu,B;\{m\})$$

- The self-consistency condition:

$$\frac{\partial P}{\partial m} = 0$$

- thus obtained mass must be a symmetric function of μ:

$$\frac{\partial m}{\partial \mu}\ |_{\mu=0} = 0$$

Energy spectra of hadrons in uniform B with κ

- For **spin-1/2** states ($g = 2Q + 2\kappa$): Exact [Tsai and Yildiz , Phys. Rev. D (1971)]

$$E_{ch} = \sqrt{\left(\sqrt{M^2 + \mathcal{B}|Q|(2l+1) - 2Q\mathcal{B}s_z}\right) - \mu_M \mathcal{B}2\kappa s_z\right)^2 + p_z^2}$$

$$E_{neu} = \sqrt{\left(\sqrt{M^2 + p^2 - p_z^2} - \mu_M \mathcal{B}2\kappa s_z\right)^2 + p_z^2}$$

- For **spin 1 and 3/2**($g = 2Q + 2\kappa$) : Good approximation [Ferrar, Phys. Rev. D (1992); Belinfante, Phys. Rev. (1953); Paoli, J. Phys. G (2013)]

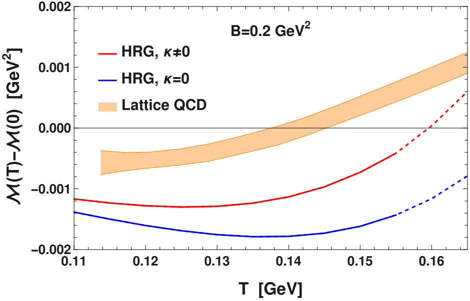
$$E_{ch} = \sqrt{M^2 + p_z^2 + \mathcal{B}|Q|(2l+1) - 2Q\mathcal{B}s_z} - \mu_M \mathcal{B}2\kappa s_z \ ,$$

$$E_{neu} = \sqrt{M^2 + p^2} - \mu_M \mathcal{B}2\kappa s_z$$

- For **spin** $>$ **3/2** ($g = 2Q + 2\kappa$): Approximation

$$E_{ch} = \sqrt{M^2 + p_z^2 + \mathcal{B}|Q|(2l+1)} - \mu_M \mathcal{B}g s_z \ , \ E_{neu} = \sqrt{M^2 + p^2} - \mu_M \mathcal{B}g s_z$$

Magnetization



Same problem !!

Pauli-Villars Regularization

- Vacuum contribution to quark:

$$\chi_B^{\text{vac}} = R_0 + R_1\kappa + R_2\kappa^2$$

where , e.g. $R_0 = \frac{4}{3}Q^2\pi^2\left[(q^2+2m^2)B_0(q^2,m^2,m^2) - 2A_0(m^2)\right]$

- A_0 and B_0 are the one-loop Passarino-Veltman (PaVe) functions

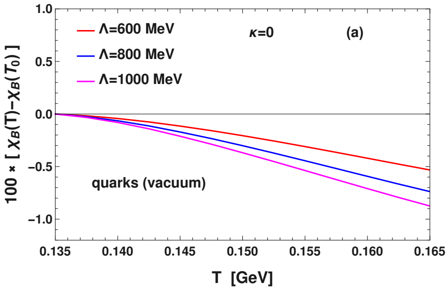
$$i\pi^2 A_0(m^2) = \int d^4p\,\frac{1}{p^2-m^2+i\epsilon}$$

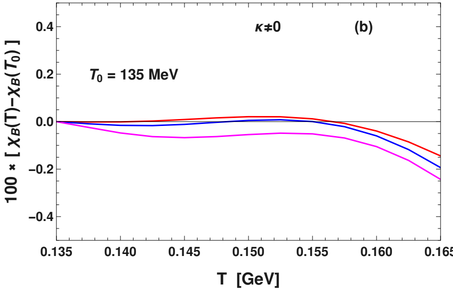
$$i\pi^2 B_0(q^2,m_1^2,m_2^2) = \int d^4p\left[\frac{1}{(p+q)^2-m_1^2+i\epsilon}\,\frac{1}{p^2-m_2^2+i\epsilon}\right]$$

- Replace $F(\{m^2\})$ with

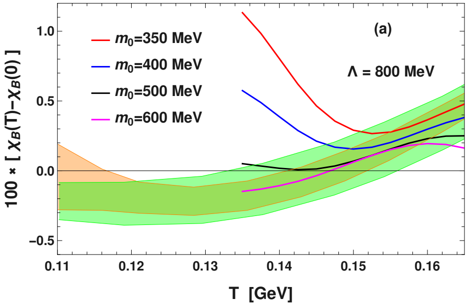
$$F^\Lambda(\{m^2\}) = F(\{m^2\}) - F(\{m^2+\Lambda^2\}) + \Lambda^2\frac{dF(\{M^2+\Lambda^2\})}{d\Lambda^2}$$

Effect of Λ





Fixing $m_q(T=0)$



We try several values of $m_l(0)$ ($m_s(0) = m_l(0) + 120$ MeV) and fix it to $m_l(0) = 600$ MeV $\longrightarrow$ best agreement with the data